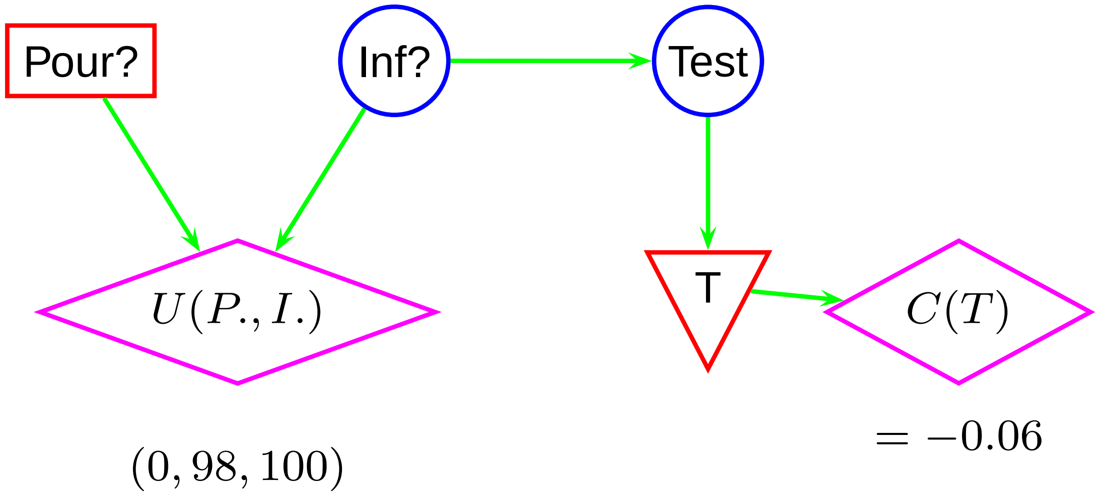
Bayesian Networks and Decision Graphs

Chapter 11

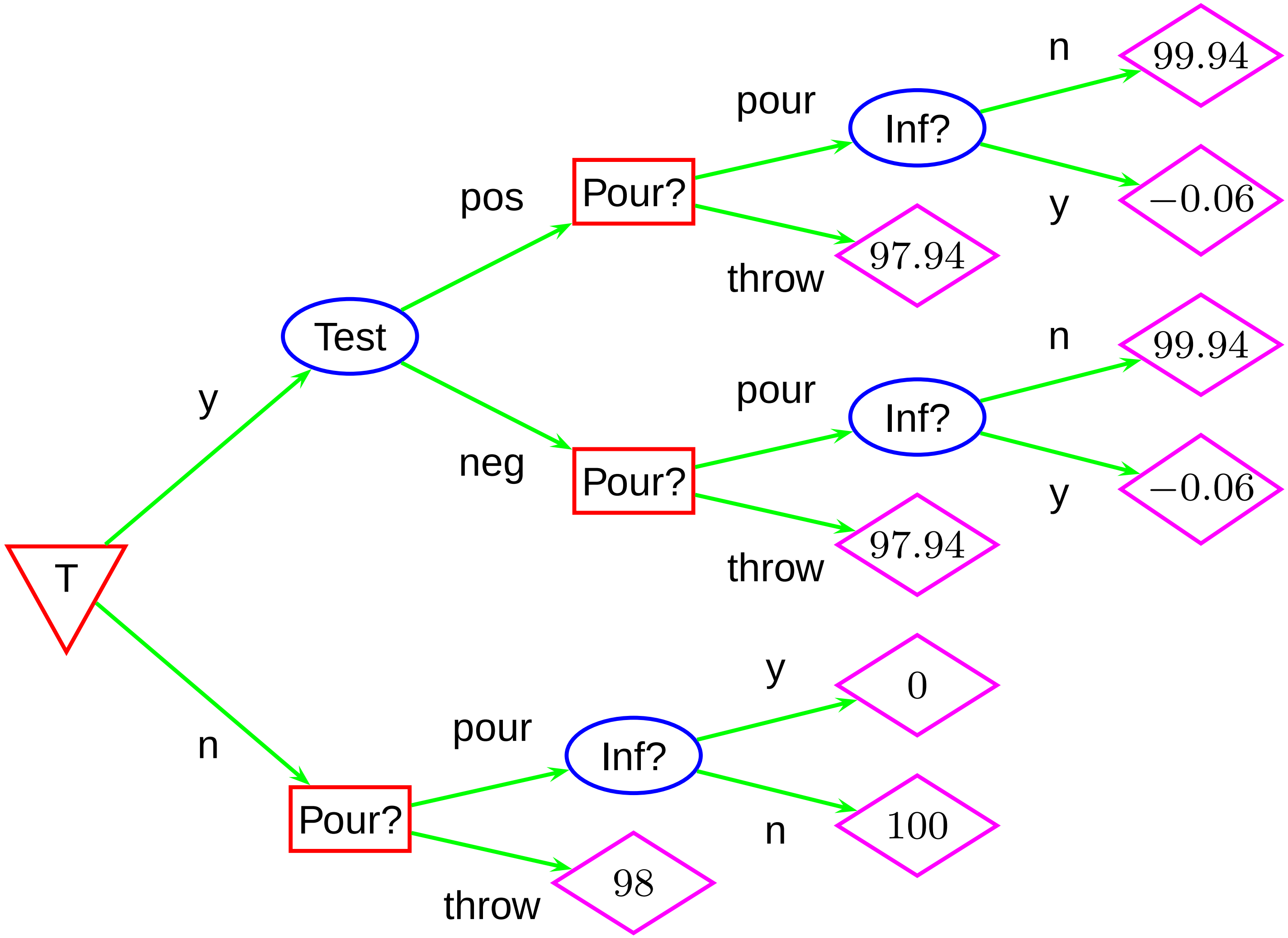
Value of information I



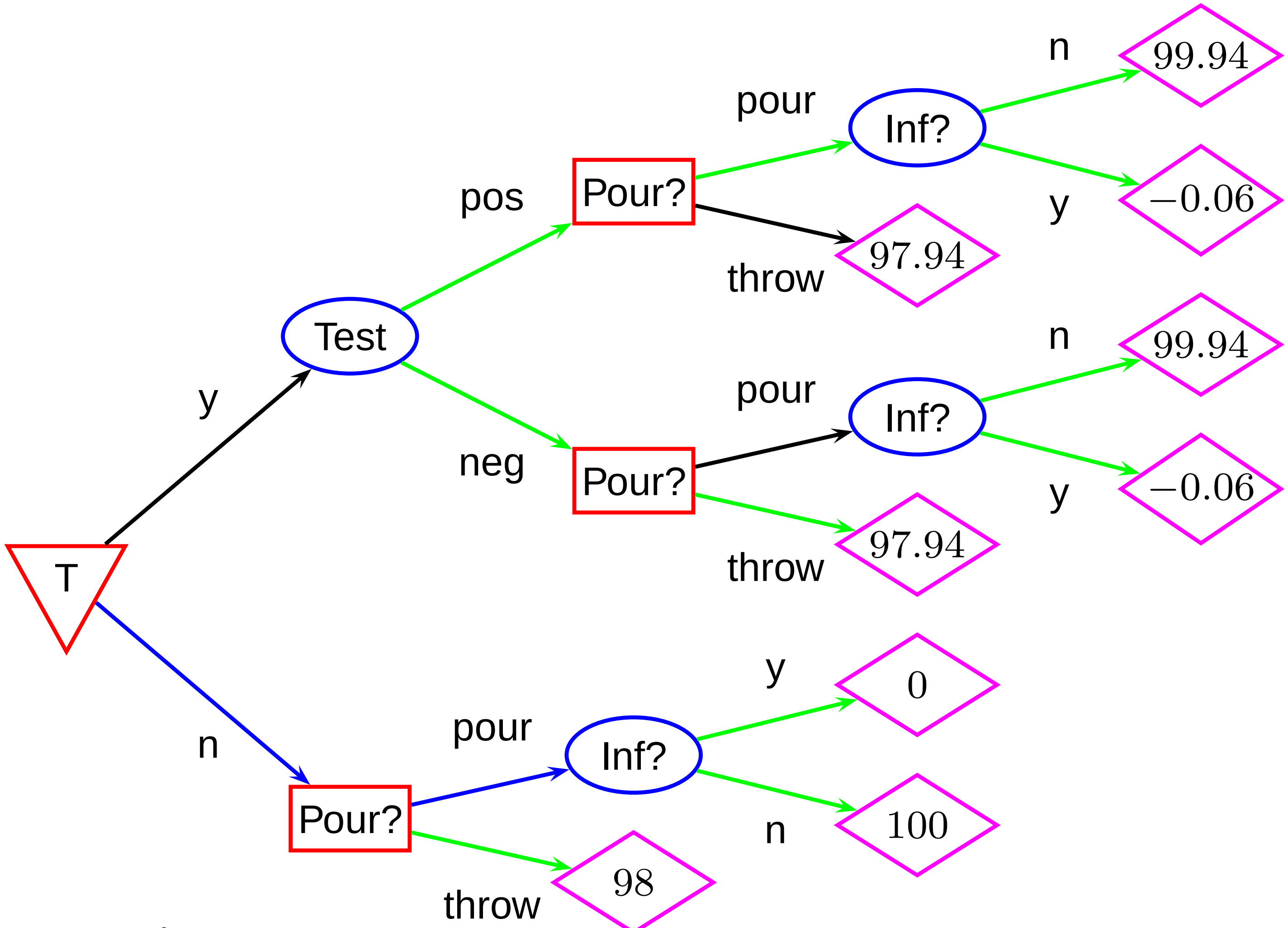
There are 50 cows, and the milk is poured into a common container.

Does it pay to perform the test??

Value of information II



Value of information II

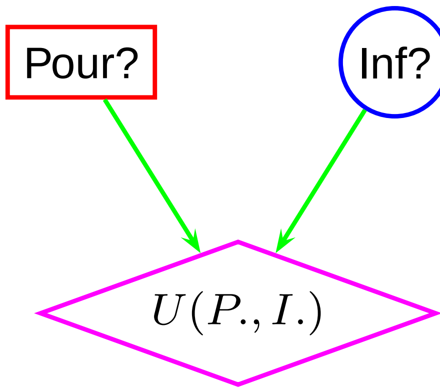


For the two strategies:

$EU(\text{blue}) = 0 \cdot P(\text{Inf?} = \text{y}) + 100 \cdot P(\text{Inf?} = \text{n}) = 99.93$

$EU(\text{black}) = 97.94 \cdot P(\text{Test} = \text{pos}) + 99.94 \cdot P(\text{Inf?} = \text{n}, \text{neg}) - 0.06 \cdot P(\text{Inf?} = \text{y}, \text{neg}) = 99.92$

Value of a distribution



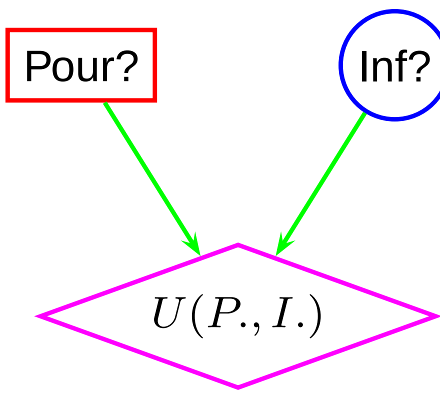
Maximal expected utility:

$$\max_{\text{Pour?}} \sum_{\text{Inf?}} U(\text{Pour?}, \text{Inf?}) P(\text{Inf?})$$

We say that the value of $P(\text{Inf?})$ is $V(P(\text{Inf?})) = \max_{\text{Pour?}} \sum_{\text{Inf?}} U(\text{Pour?}, \text{Inf?}) P(\text{Inf?})$.

Moreover, the value after a positive test result is $V(P(\text{Inf?} | \text{pos}))$.

Value of a distribution



Maximal expected utility:

$$\max_{\text{Pour?}} \sum_{\text{Inf?}} U(\text{Pour?}, \text{Inf?}) P(\text{Inf?})$$

We say that the <u>value</u> of $P(\text{Inf?})$ is $V(P(\text{Inf?})) = \max_{\text{Pour?}} \sum_{\text{Inf?}} U(\text{Pour?}, \text{Inf?}) P(\text{Inf?})$.

Moreover, the value after a positive test result is $V(P(\text{Inf?}|\text{pos}))$.

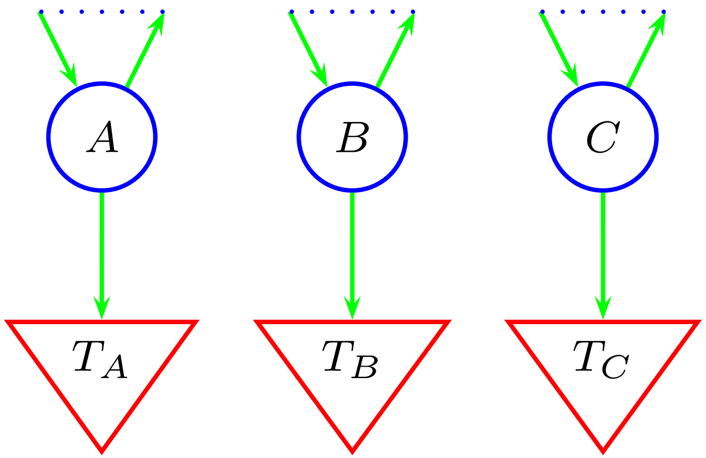
The expected value after Test:

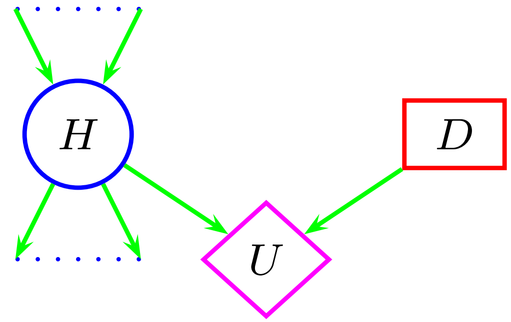
$$EV(\text{Test}) = V(P(\text{Inf?}|\text{pos}))P(\text{pos}) + V(P(\text{Inf?}|\text{neg}))P(\text{neg}).$$

<u>Notation:</u>

- Expected benefit: $EB(\text{Test}) = EV(\text{Test}) - V(P(\text{Inf?}))$
- Expected profit: $EP(\text{Test}) = EB(\text{Test}) - C_{\text{Test}}$.

Hypothesis driven, myopic





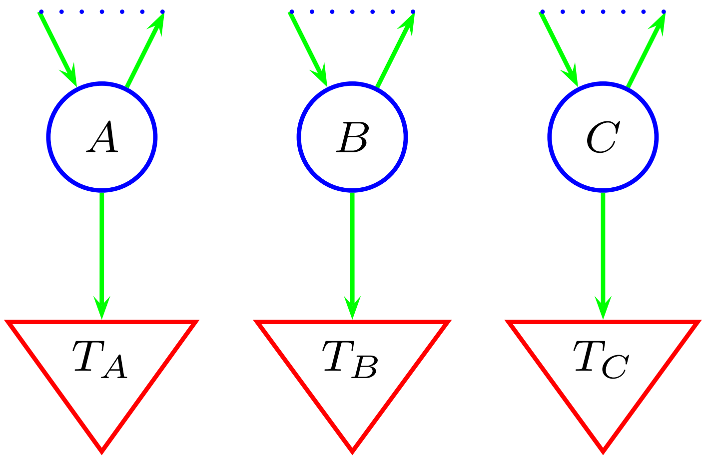
If T yields result t, then:

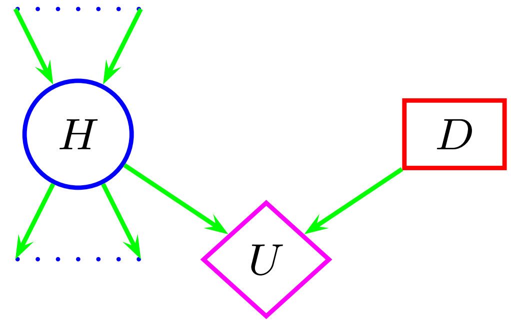
$$V(P(H|t)) = \max_D \sum_H U(D, H)P(H|t)$$

Expected value after T: $\sum_T V(P(H|T))P(T)$

So we request: $P(T)$ and $P(H|T)$ for all T!

Hypothesis driven, myopic





If T yields result t, then:

$$V(P(H|t)) = \max_D \sum_H U(D,H)P(H|t)$$

Expected value after T: $\sum_T V(P(H|T))P(T)$

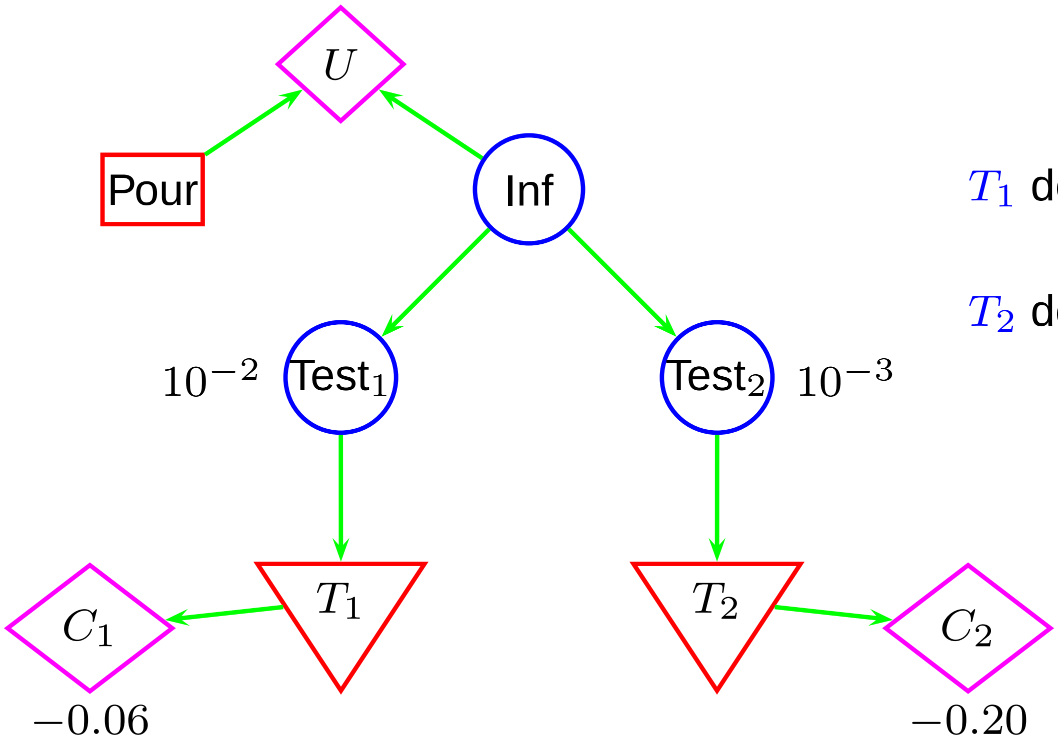
So we request: $P(T)$ and $P(H|T)$ for all T!

From Bayes' rule:

$$P(H|T) = \frac{P(T|H)P(H)}{P(T)}$$

One propagation for each state of H suffices!!

Non-myopic data request I



T_1 does not pay

T_2 does not pay

It does not pay to perform test T_2 alone, however, what if we use T_2 to double check a positive result from test T_1?

Value functions without utilities

If there is no proper model for actions and utilities, we may look at:

- Entropy
- Variance
- Other

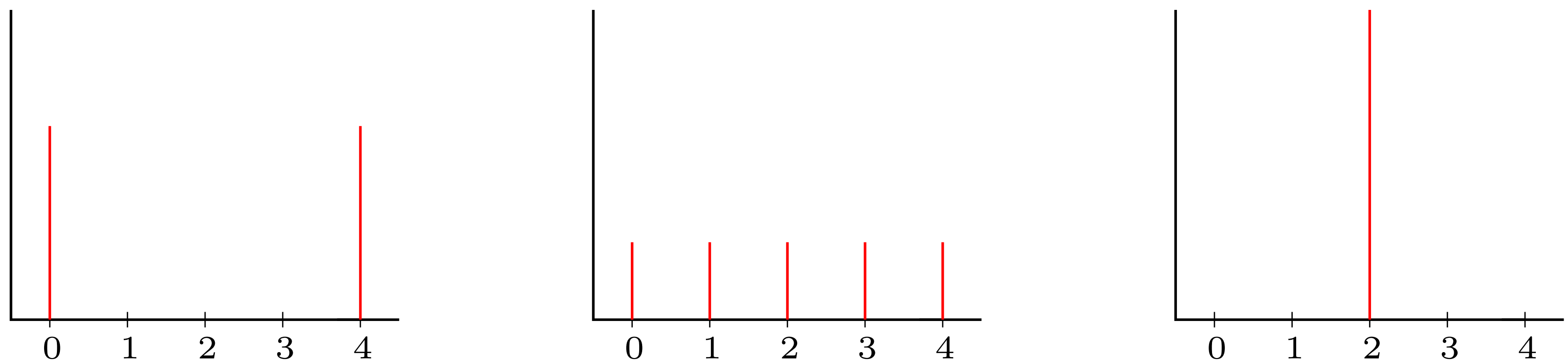
We require that:

- △ The expected change in value may not decrease.

Variance

If the states of H are numeric, then:

$$\mu = \sum_{h \in H} h \cdot P(h) \qquad Var(P(H)) = \sum_{h \in H} (h - \mu)^2 P(h)$$



$\mu = 2; Var = 4$ $\mu = 2; Var = 2$ $\mu = 2; Var = 0$

Now let the value function be:

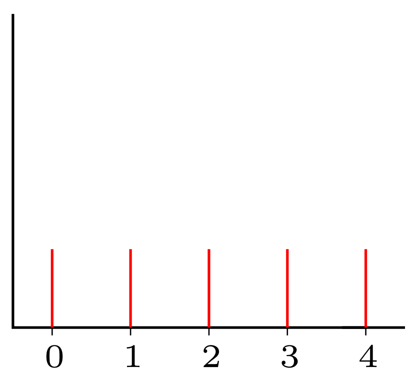
$$V(P(H)) = -Var(P(H))$$

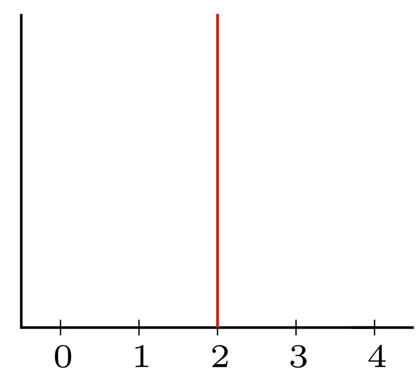
Entropy

The entropy is defined as:

$$Ent(P(H)) = -\sum_H P(H)\log_2(P(H)) \qquad (0 \cdot \log_2(0))$$

Entropy is a measure of how much the probability mass is scattered around on the states.





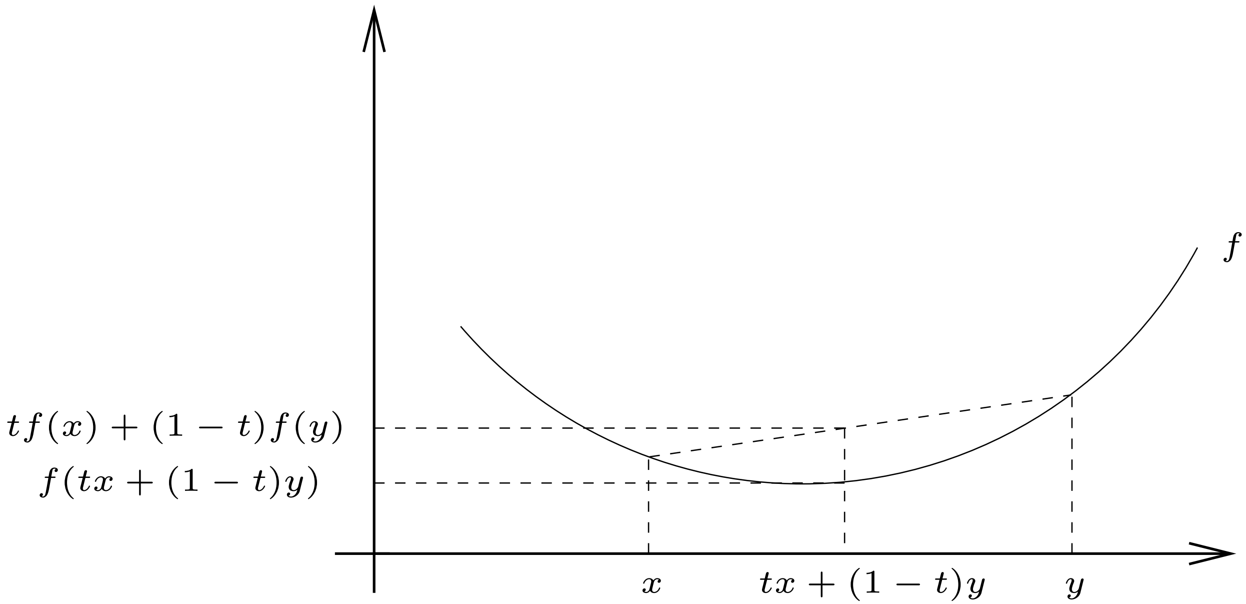
$$Ent(P(H)) = -\sum \frac{1}{n}\log_2 \frac{1}{n} = \log_2 n = 2.3$$

$$Ent(P(H)) = 0$$

<u>We let the value function be:</u>

$$V(P(H)) = -Ent(P(H)) = \sum_H P(H)\log_2(P(H))$$

Other value functions



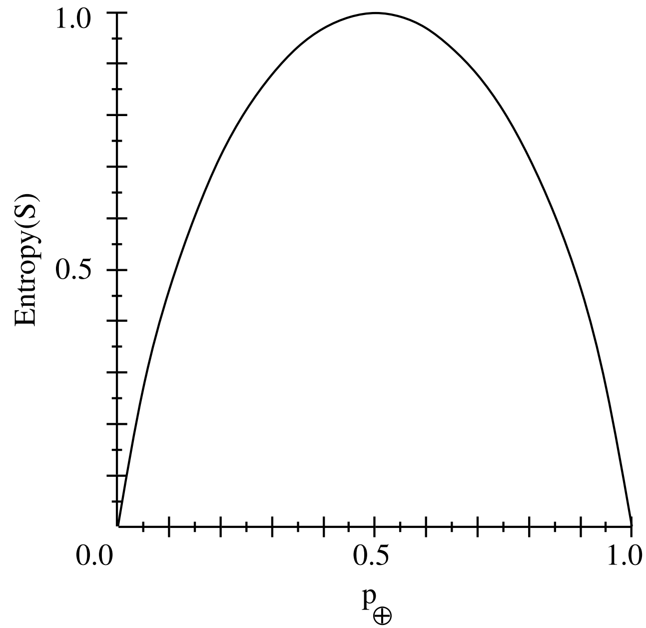
$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y)$$

Theorem: If the value function is convex, then the expected benefit of a test is never negative.

Note that: Value functions based on entropy, variance and max-utility are convex.

Entropy

The entropy function of a binary variable:



$$Ent(P(H)) = -P(yes)\log_2(P(yes)) - P(no)\log_2(P(no))$$

From this we see that the value function based on entropy is convex!