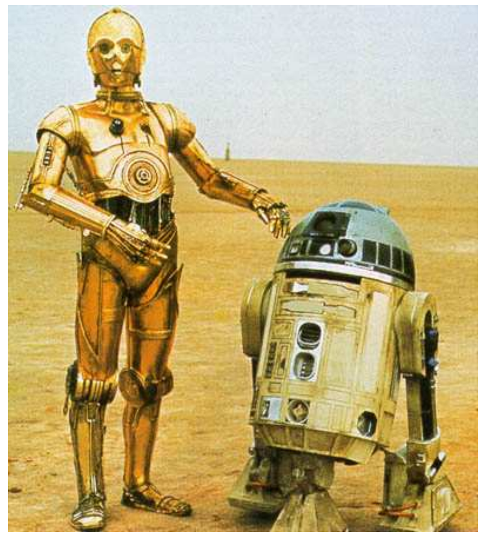
Bayesian Networks and Decision Graphs

Chapter 2

The grand vision

An autonomous self-moving machine that acts and reasons like a human



We are still very far away from achieving this goal!

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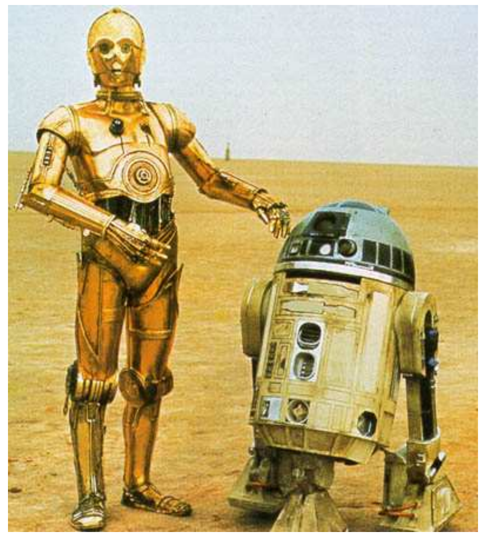

We are still very far away from achieving this goal!

Research is going in two directions:

- Robotics
- Artificial intelligence

A recent achievement: DARPA grand challenge 2005

Competition for autonomous vehicles: navigate 132 miles through desert terrain (route specified by approx. 3000 “waypoints”). 5 out of 23 vehicles completed the task. Winner: Stanley of Stanford Racing Team in 6h 53m (19.2 mph).

- 7 Pentium M computers
- Sensors: 4 laser range finders, 1 radar system, 1 stereo camera pair, 1 monocular vision system, GPS, inertial measurement unit, wheel speed.

Robotics

Tasks:

- Visual recognition of objects
- Recognition of sound patterns
- Balancing (to walk with n legs)
- Positioning in space
- ...

Criteria of success: Real time movement in space.

Robotics

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Scientifically and computationally extremely demanding, however:

- basically you construct a machine that behaves like an animal (dog, ant, etc.)

Artificial intelligence

Areas:

- Complex arithmetic
- Reduction of mathematical expressions
- Computations
- Games (chess)
- Type setting

Computers can do (much of) this—they do not really require artificial intelligence!

A particular branch of AI has to do with reasoning

- E.g. Logical reasoning (Boolean algebra and its algorithms).

When a task is understood so much that it can be formalized, then it is no longer considered intelligent.

Boolean logic

Examples:

$$\frac{\text{It rains} \Rightarrow \text{The grass is wet}\ ,\ \text{It rains}}{\text{The grass is wet}}$$

$$\frac{\text{It rains} \Rightarrow \text{The grass is wet}\ ,\ \text{The grass is not wet}}{\text{It does not rain}}$$

What if there is uncertainty?

- "If I take a cup of coffee in the break, then I may stay awake during the next lecture".

Uncertainty can appear and be expressed in several ways:

- Fuzzy concepts (large, heavy, pretty)
- Uncertain information
- Non-deterministic relations
  - Disease $\longrightarrow$ Symptoms
  - Treatment $\longrightarrow$ Result
- Incomplete knowledge/information

Reasoning under uncertainty

Imagine that we extend Boolean algebra with certainty $x \in [0; 1]$.

- "A holds with certainty x".

Combination:

- I take a cup of coffee in the break $\rightarrow_{0.5}$ I will stay awake
- I take a walk in the break $\rightarrow_{0.8}$ I will stay awake

Suppose that I take a walk as well as have a cup of coffee. Then:

- I stay awake with certainty $f(0.5, 0.8)$

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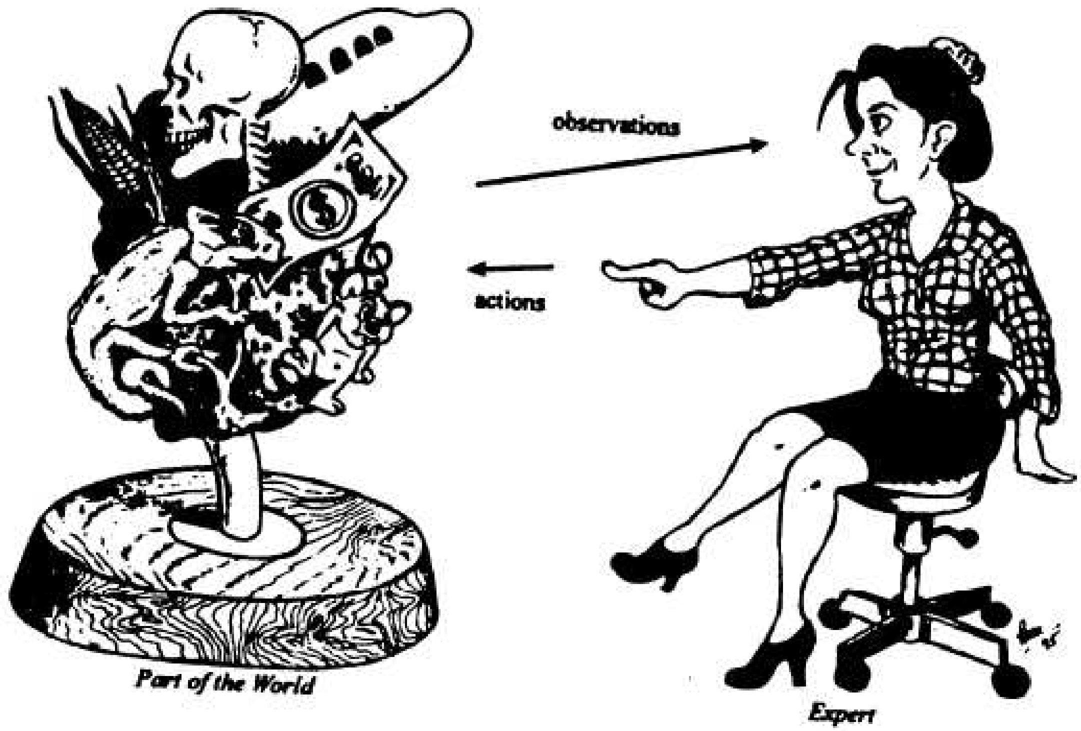
Chaining:

$$\frac{a \rightarrow_x b \quad b \rightarrow_y c \;, a}{c \text{ with certainty } g(x, y)}$$

Abduction:

$$\frac{\text{woman} \rightarrow_{0.8} \text{long hair} \;, \text{long hair}}{\text{woman with certainty ??}}$$

Human wisdom



Apply accumulated and processed experience

Interpretation → Intervention → Learning → Interpretation

Car start problem

In the morning, my car will not start. The start engine turns, but nothing happens. The battery is OK. The problem may be due to dirty spark plugs or the fuel may be stolen. I look at the fuel meter. It shows $\frac{1}{2}$, and I therefore expect the spark plugs to be dirty.

We need to formalize this kind of reasoning:

- What made me focus upon fuel and spark plugs?
- Why did I look at the fuel meter?
- Why had fuel meter reading an impact on my belief in dirty spark plugs?

The car start problem (causally)

Events:

- Fuel?{y,n}
- Clean spark plugs?{y,n}
- Start?{y,n}
- Fuel meter{full,$\frac{1}{2}$,empty}.

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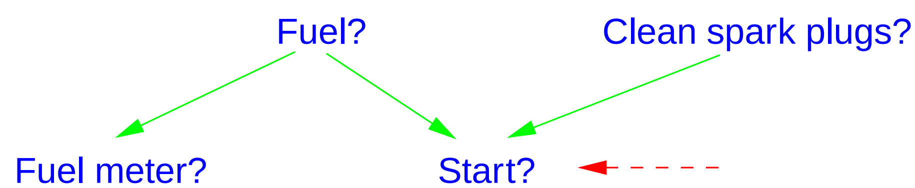
Events:

- Fuel?{y,n}
- Clean spark plugs?{y,n}
- Start?{y,n}
- Fuel meter{full,$\frac{1}{2}$,empty}.

Causal relations:



When I enter the car I have some prior belief on the various events but then start=n.

Direction change of belief

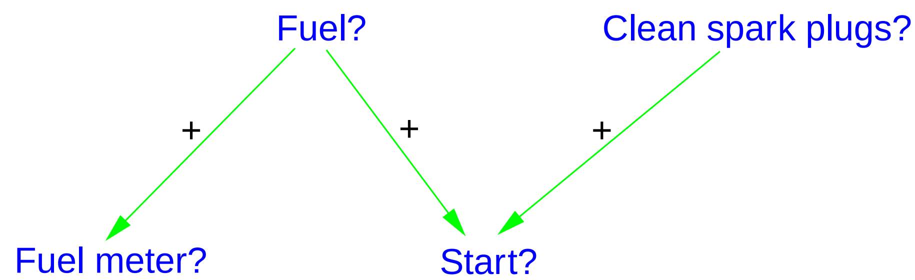
Call:

- the direction from n to y positive.
- the direction more fuel positive.

$$\text{Fuel?}_{\rightarrow +} \rightarrow \text{Fuel meter}_{\rightarrow +} \quad \Rightarrow \quad \text{Fuel?} \rightarrow_{+} \text{Fuel meter}$$

Note: $\text{Fuel meter}_{\rightarrow +} \Rightarrow \text{Fuel?}_{\rightarrow +}$

The reasoning



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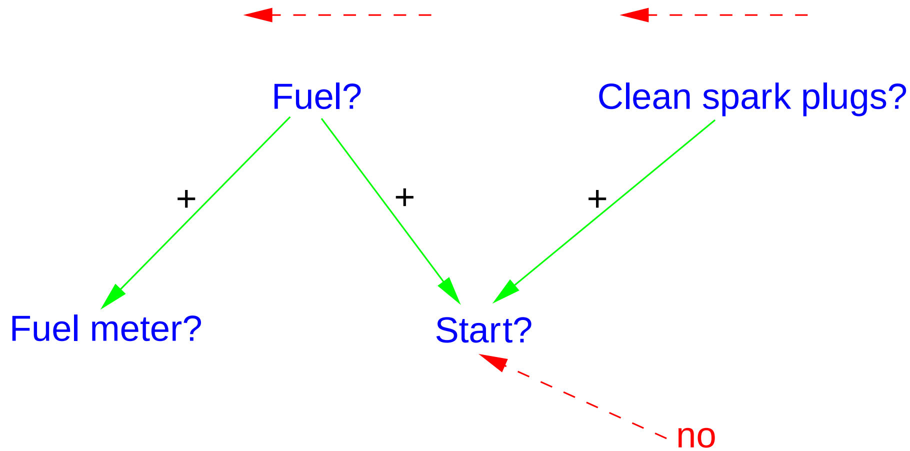
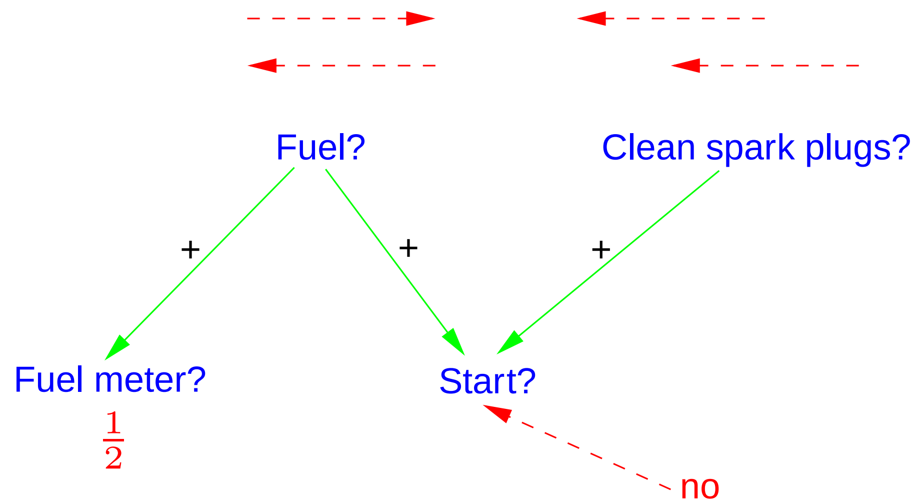
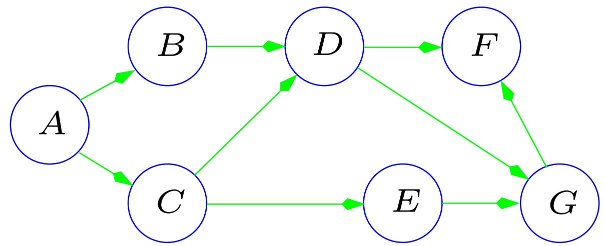


The reasoning



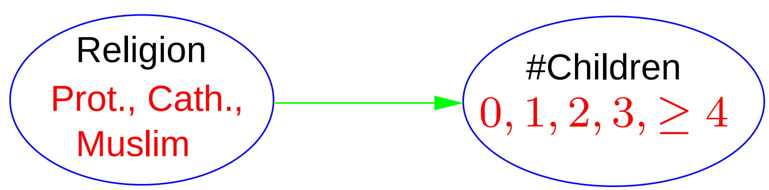
Causal networks

A causal network is a directed acyclic graph:



- The nodes are variables with a finite set of states that are mutually exclusive and exhaustive:
  - For example {y,n}, {red, blue, green}, {0,1,2,3,42}.
- The links represent cause – effect relations.

For example:

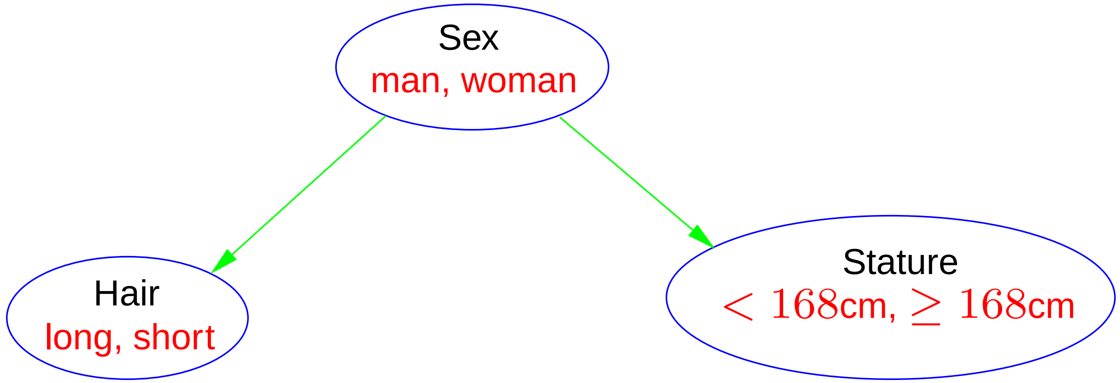


All variables are in exactly one state, but we may not know which one.

Reasoning under uncertainty 1

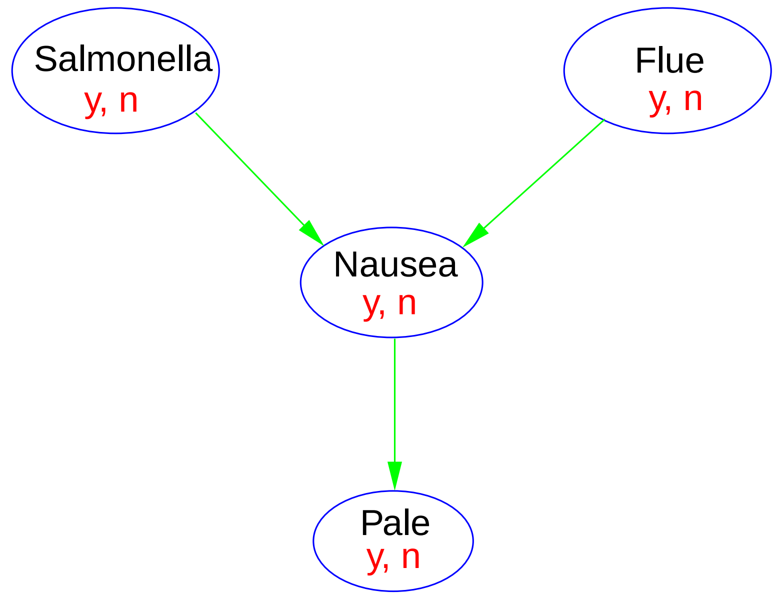
- If there has been a flooding does that tell me something about the amount of rain that has fallen?
- The water level is high: If there has been a flooding does that tell me anything new about the amount of rain that has fallen?

Reasoning under uncertainty 2



- If a person has long hair does that say something about his/her stature?
- It is a woman: If she has long hair does that say something about her stature?

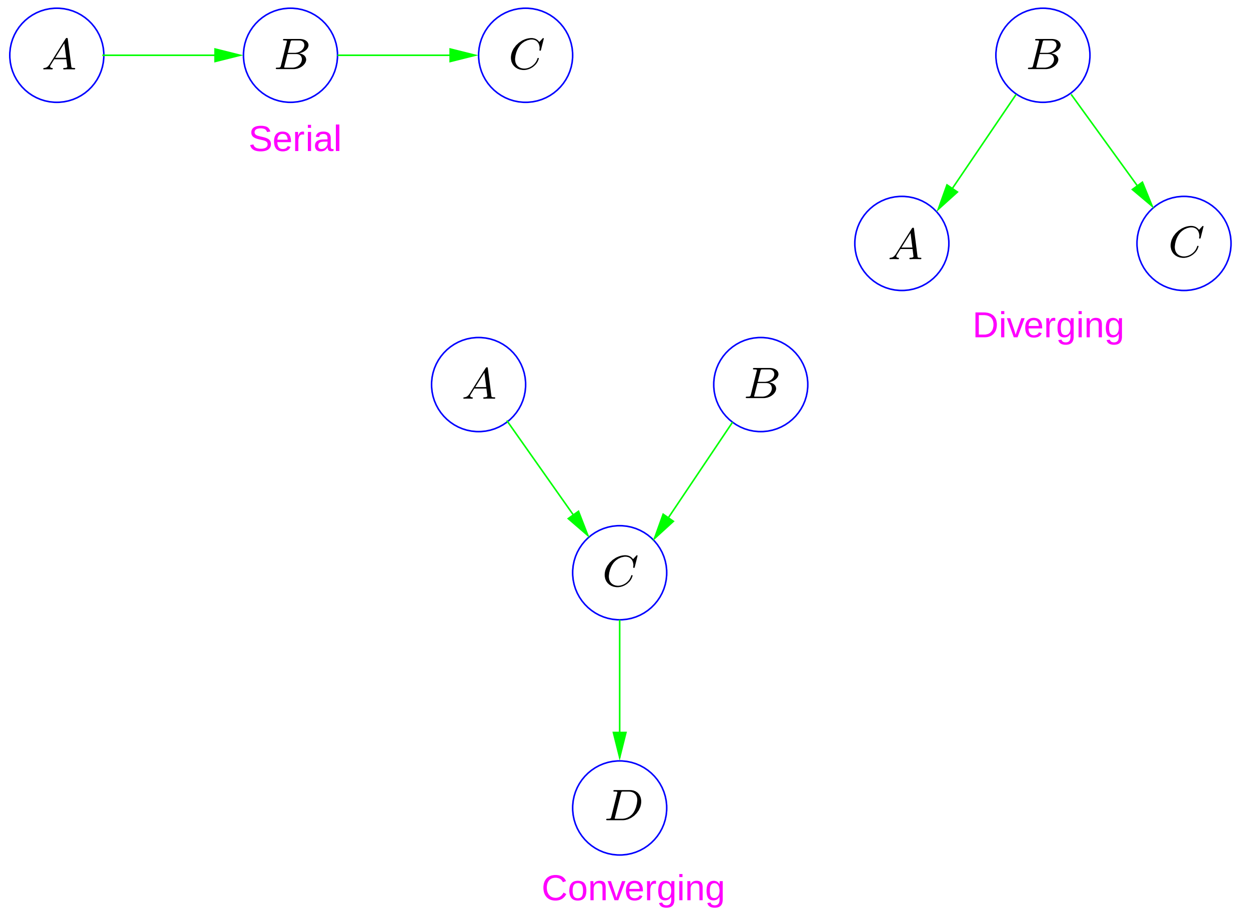
Reasoning under uncertainty 3



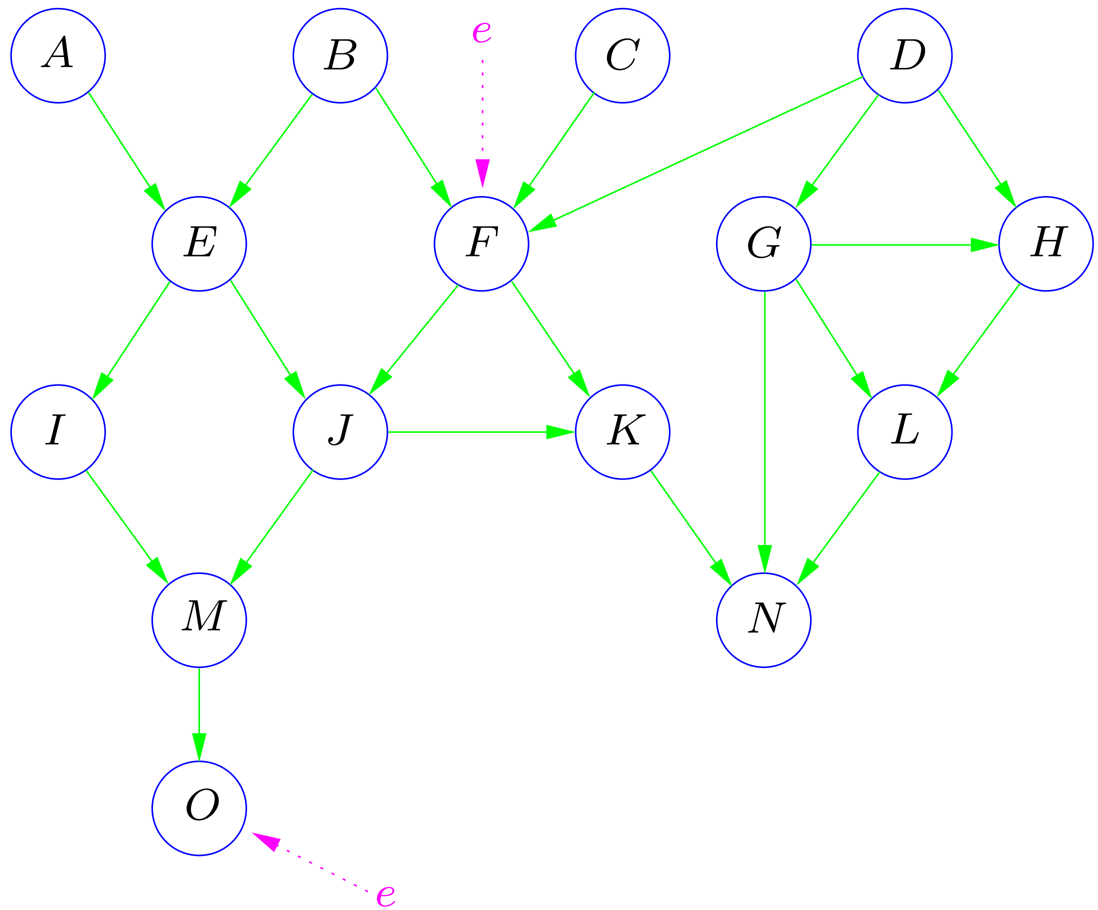
- Does salmonella have an impact on Flue?
- If a person is Pale, does salmonella then have an impact on Flue?

Transmission of evidence 1

Relevance changes with evidence

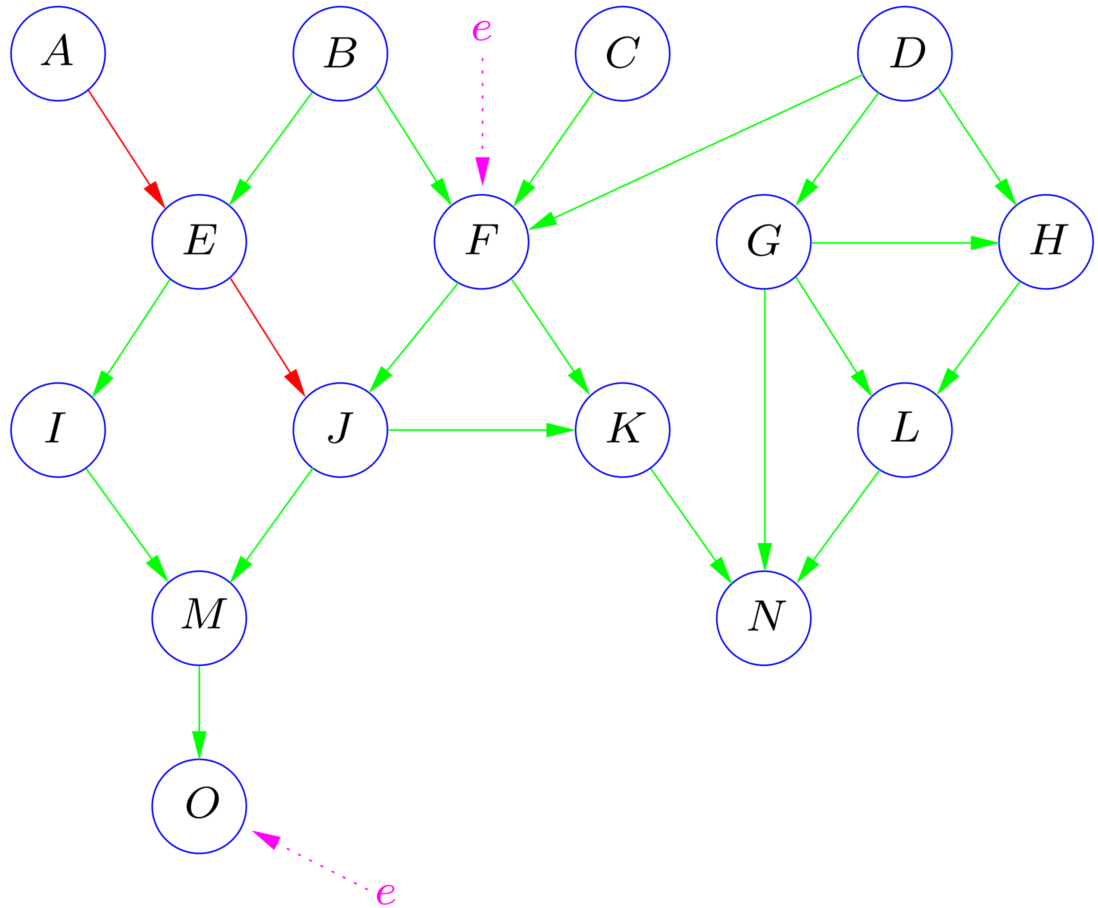


Transmission of evidence 2



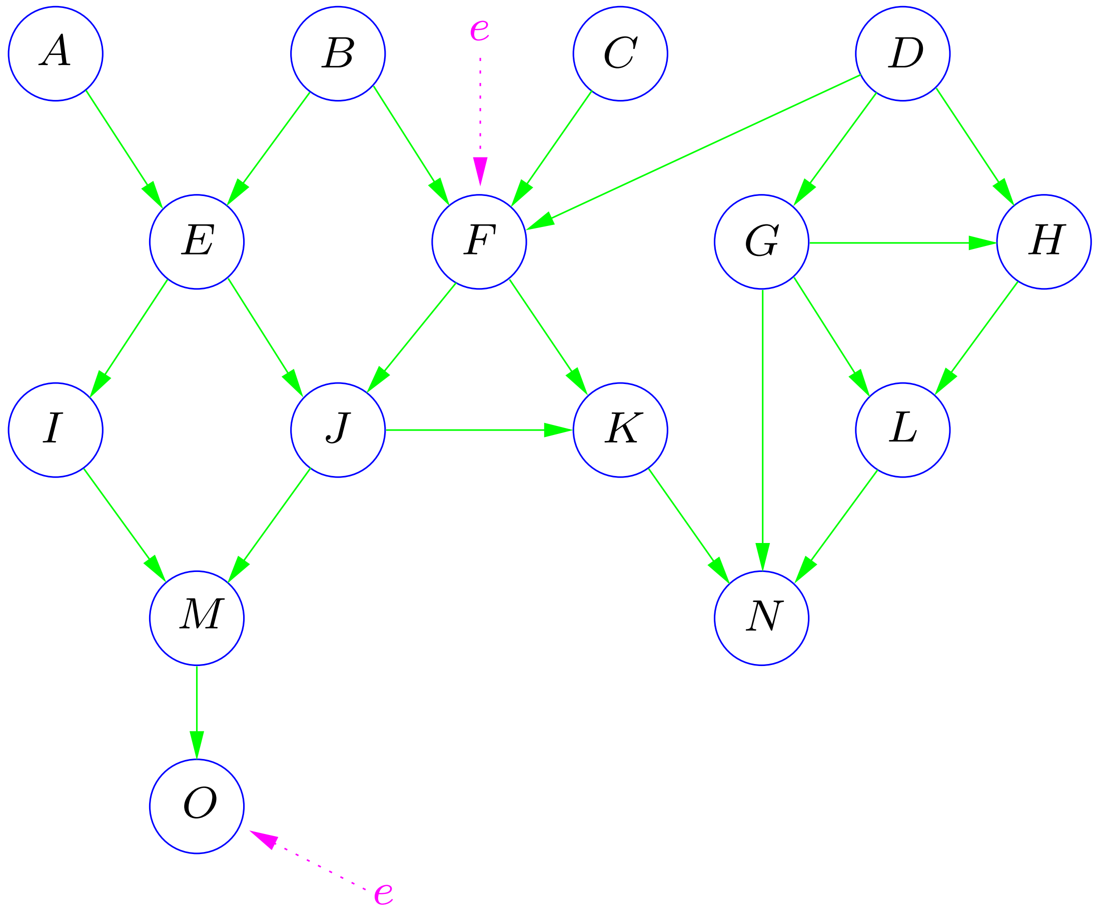
Can knowledge of A have an impact on our knowledge of J?

Transmission of evidence 2



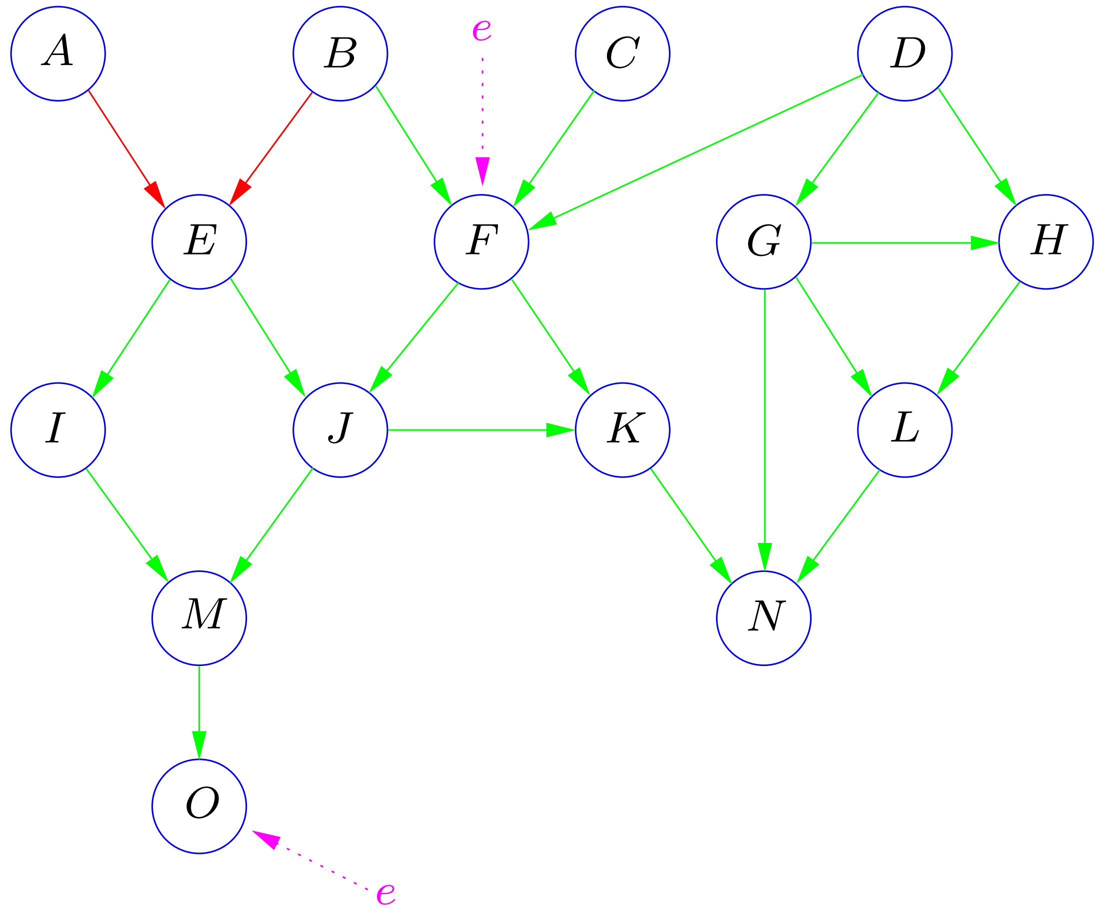
Can knowledge of A have an impact on our knowledge of J? yes!

Transmission of evidence 2



Can knowledge of A have an impact on our knowledge of B?

Transmission of evidence 2



Can knowledge of A have an impact on our knowledge of B? yes!

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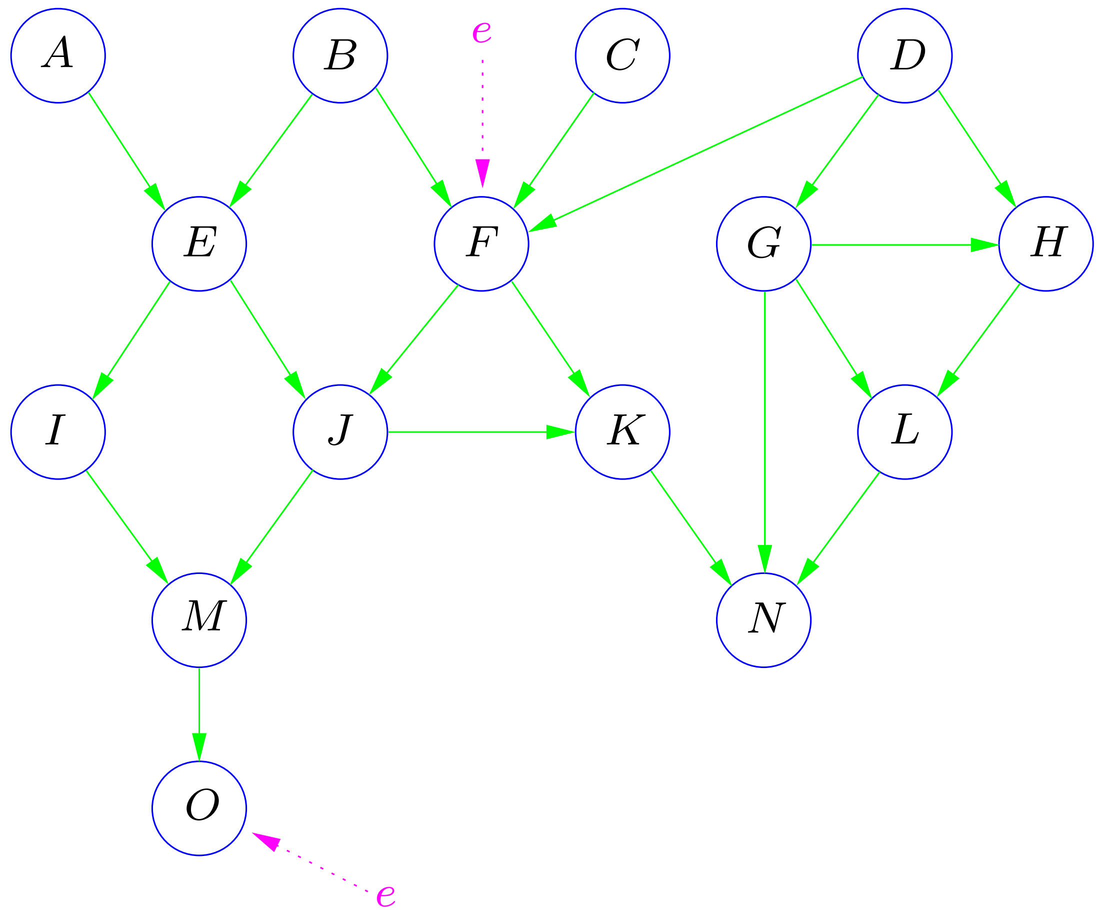
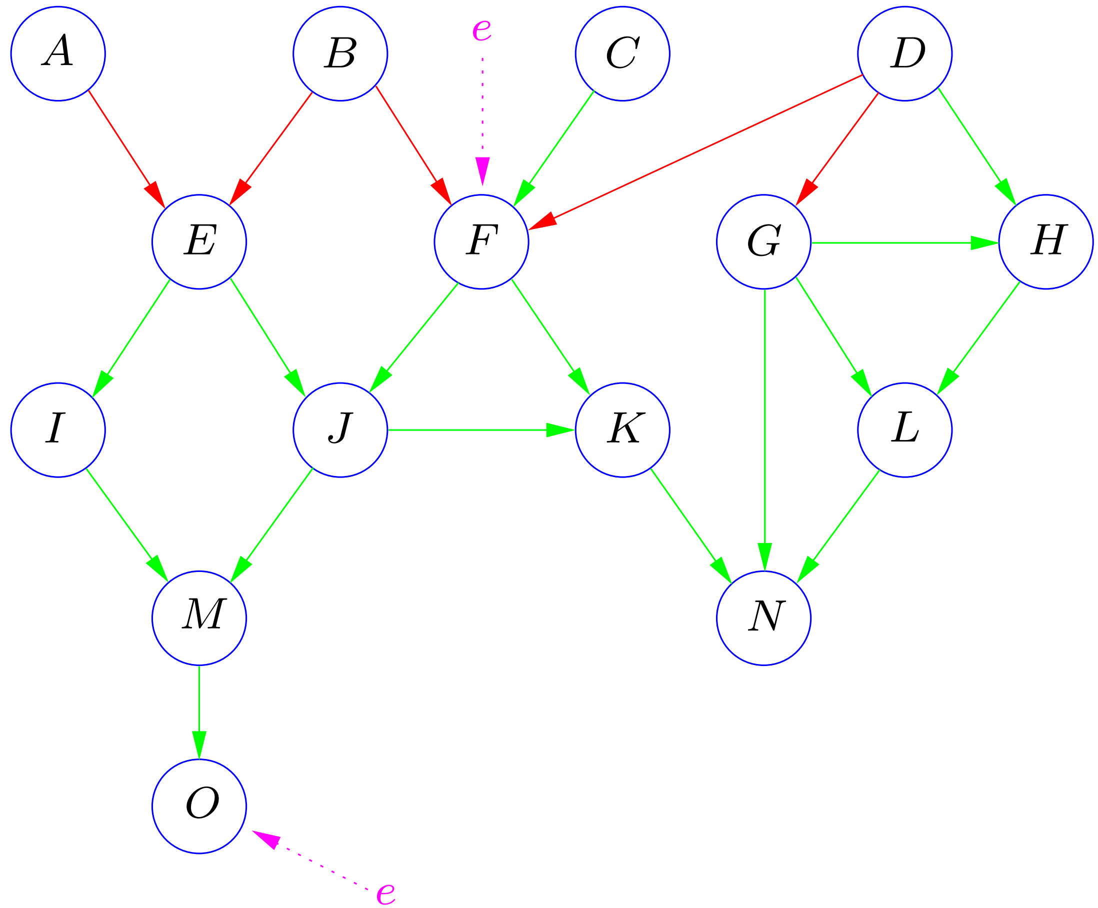


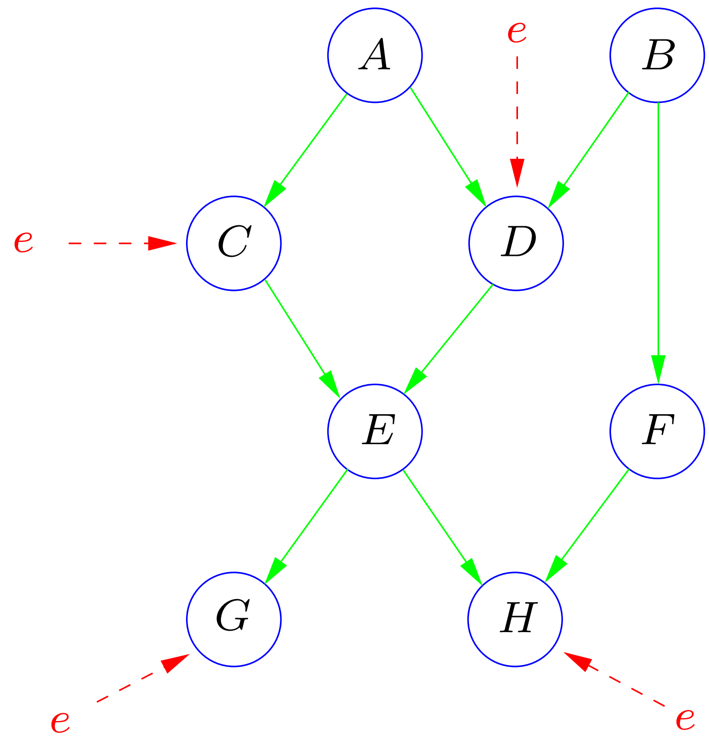
Can knowledge of A have an impact on our knowledge of G?

Transmission of evidence 2



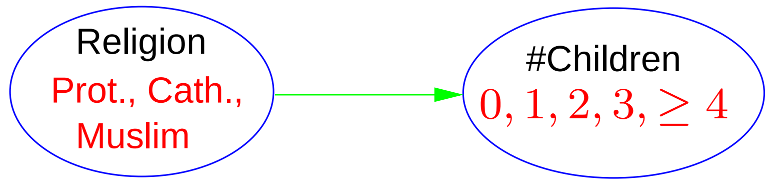
Can knowledge of A have an impact on our knowledge of G? yes!

Transmission of evidence 3



Is E d-separated from A?

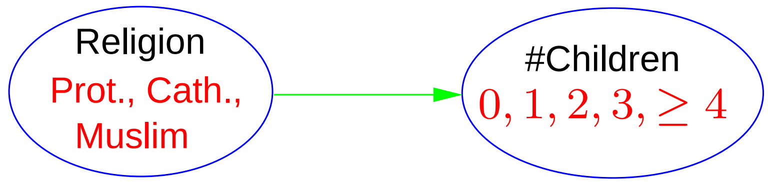
Quantification of causal networks



The strength of the link is represented by probabilities:

$P(0 \mid p)$	$P(0 \mid c)$	$P(0 \mid m)$
$P(1 \mid p)$	$P(1 \mid c)$	$P(1 \mid m)$
$P(2 \mid p)$	$P(2 \mid c)$	$P(2 \mid m)$
$P(3 \mid p)$	$P(3 \mid c)$	$P(3 \mid m)$
$P(\geq 4 \mid p)$	$P(\geq 4 \mid c)$	$P(\geq 4 \mid m)$

Quantification of causal networks



The strength of the link is represented by probabilities:

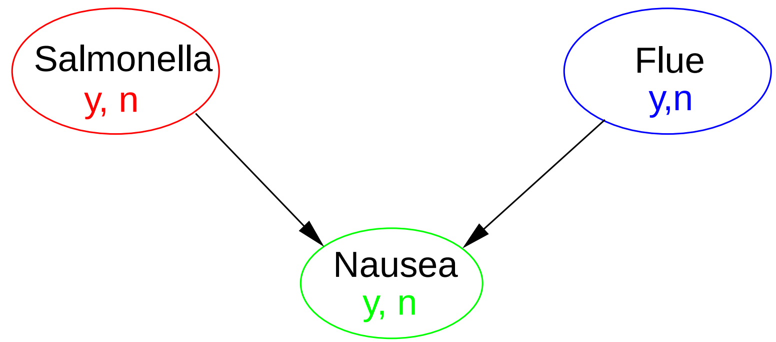
$P(0\|p)$	$P(0\|c)$	$P(0\|m)$
$P(1\|p)$	$P(1\|c)$	$P(1\|m)$
$P(2\|p)$	$P(2\|c)$	$P(2\|m)$
$P(3\|p)$	$P(3\|c)$	$P(3\|m)$
$P(\geq 4\|p)$	$P(\geq 4\|c)$	$P(\geq 4\|m)$

$\Rightarrow$

#Children	Religion: p	c	m
0	0.15	0.05	0.05
1	0.2	0.1	0.1
2	0.4	0.2	0.1
3	0.2	0.4	0.1
≥ 4	0.05	0.25	0.35

P(#Children|Religion)

Several parents



$P(y|y,y)$ $P(y|y,n)$
$P(n|y,y)$ $P(n|y,n)$
$P(y|n,y)$ $P(y|n,n)$
$P(n|n,y)$ $P(n|n,n)$

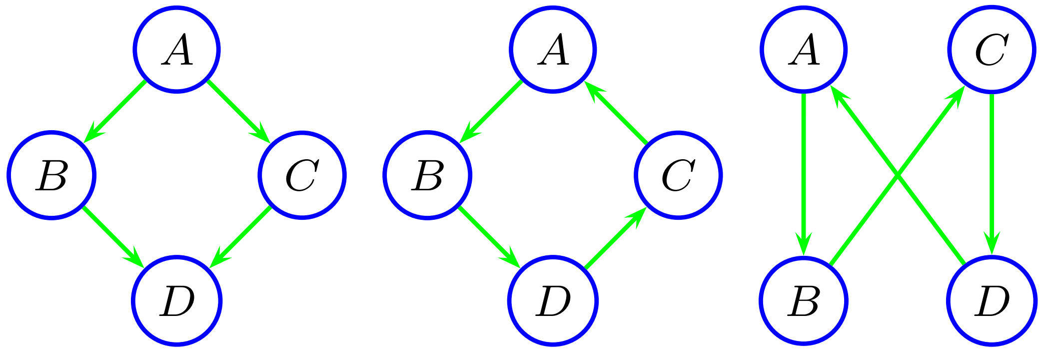
$\Rightarrow$

		Salmonella	
		y	n
Flue	y	$(0.9, 0.1)$	$(0.6, 0.4)$
	n	$(0.8, 0.2)$	$(0.1, 0.9)$

P(Nausea|Salmonella,Flue)

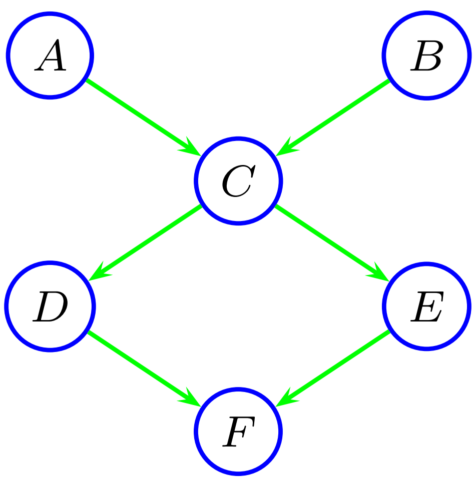
Bayesian networks

A causal network without directed cycles:



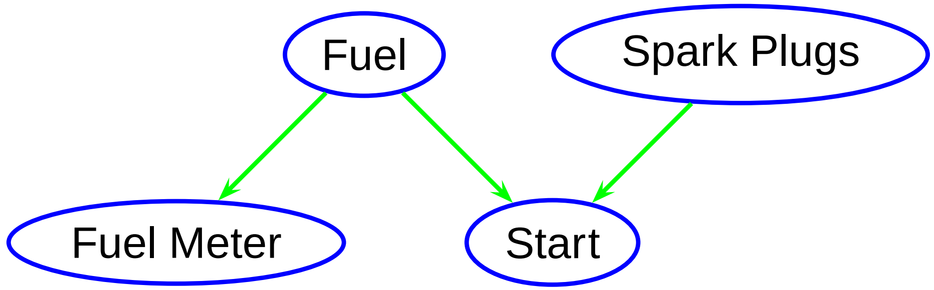
OK ¬OK ¬OK

For each variable A with parents $B_1, \ldots, B_n$ there is a conditional probability table $P(A|B_1, \ldots, B_n)$.

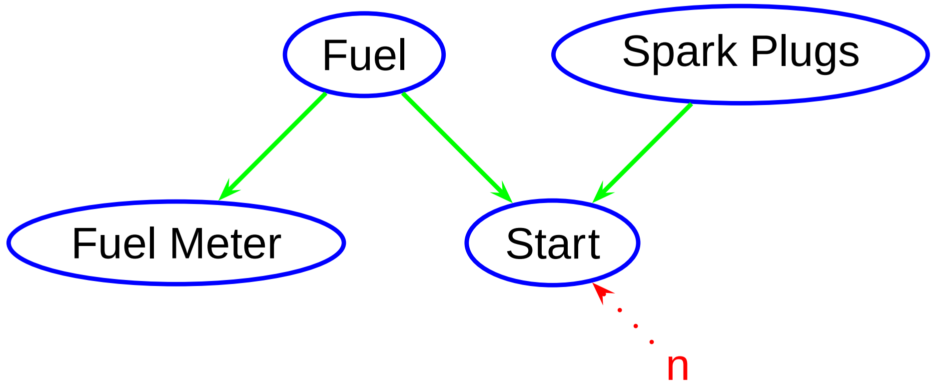


Note: Nodes without parents receive a prior distribution.

Belief updating in Bayesian networks I



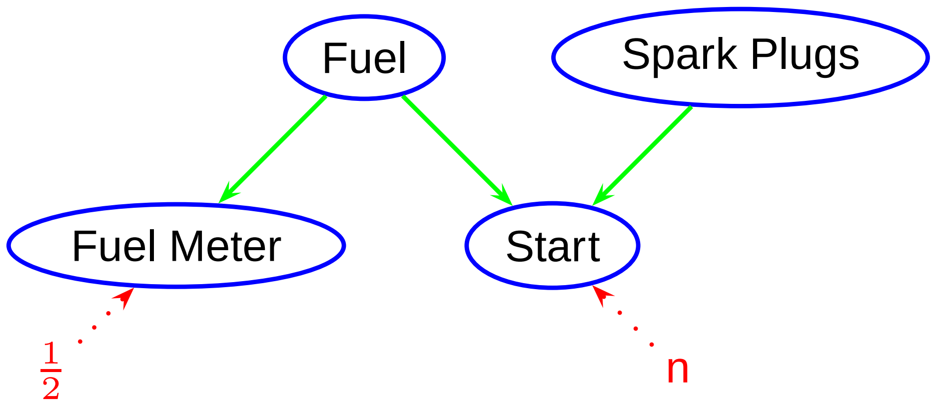
Belief updating in Bayesian networks I



Consider evidence e_1 =(Start=n) and find:

- $P($Spark Plugs$|e_1) = ??$
- $P($Fuel$|e_1) = ??$
- $P($Fuel Meter$|e_1) = ??$

Belief updating in Bayesian networks I



Consider evidence e_1 =(Start=n) and find:

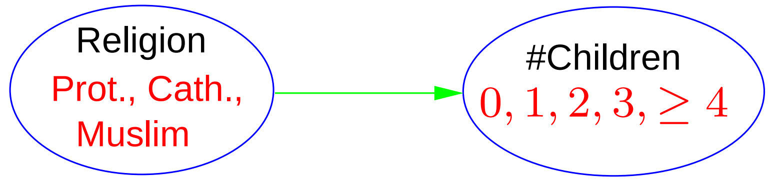
- $P($Spark Plugs$|e_1) =??$
- $P($Fuel$|e_1) =??$
- $P($Fuel Meter$|e_1) =??$

If we also have evidence e_2=(Fuel Meter $= \frac{1}{2}$) what is:

- $P($Spark Plugs$|e_1, e_2) =??$
- $P($Fuel$|e_1, e_2) =??$

Belief updating in Bayesian networks II

Consider again the network:



Assume that the probabilities are: $P(\text{Religion}) = (0.9_{\text{p}}, 0.04_{\text{c}}, 0.06_{\text{m}})$ and

		Religion		
		p	c	m
#Children	0	0.15	0.05	0.05
	1	0.2	0.1	0.1
	2	0.4	0.2	0.1
	3	0.2	0.4	0.4
	≥ 4	0.05	0.25	0.35

$P(\text{\#Children}|\text{Religion})$

$\Rightarrow$

		Religion		
		p	c	m
#Children	0	0.135	0.002	0.003
	1	0.18	0.004	0.006
	2	0.36	0.008	0.006
	3	0.18	0.016	0.024
	≥ 4	0.045	0.01	0.021

$P(\text{\#Children}, \text{Religion})$

We want the probability $P(\text{Religion}|\text{\#Children} = 3)$!

Belief updating in Bayesian networks II

Let A, B and C be variables.

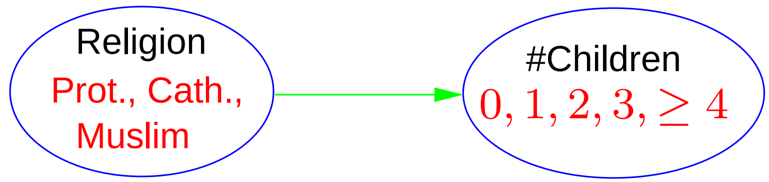
The fundamental rule: $P(A, B) = P(A|B)P(B)$.

Marginalization: $P(A) = \sum_B P(A, B)$

Bayes rule:

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

Belief updating in Bayesian networks II



We can compute $P(\text{Religion}|\#\text{Children} = 3)$ using Bayes' rule:

$$P(\text{Religion}|\#\text{Children} = 3) = \frac{P\left(\#\text{Children} = 3|\text{Religion}\right) P(\text{Religion})}{\sum_{\text{Religion}} P(\text{Religion}, \#\text{Children} = 3)}$$

#Children		Religion p	Religion c	Religion m
	0	0.135	0.002	0.003
	1	0.18	0.004	0.006
	2	0.36	0.008	0.006
	3	0.18	0.016	0.024
	≥ 4	0.045	0.01	0.021

$P(\#\text{Children}, \text{Religion})$

Conditioning $\Rightarrow$

	Religion p	Religion c	Religion m
#Ch=3	0.82	0.07	0.11

$P(\text{Religion}|\#\text{Children} = 3)$

Normalization

Consider the joint probability table $P(A, B, C)$:

		B		
		b_1	b_2	b_3
A	a_1	$(0, 0.05, 0.05)$	$(0.05, 0.05, 0)$	$(0.05, 0.05, 0.05)$
	a_2	$(0.1, 0.1, 0)$	$(0.1, 0, 0.1)$	$(0.2, 0, 0.05)$

Assume evidence e: $A = a_2$ and $C = c_1$.

What is:

$$P(B, e) =??$$

$$P(B|e) = \frac{P(B, e)}{P(e)} =??$$

Normalization

Consider the joint probability table $P(A, B, C)$:

		B		
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	a_2	$(0.1, 0.1, 0)$	$(0.1, 0, 0.1)$	$(0.2, 0, 0.05)$

Assume evidence e: $A = a_2$ and $C = c_1$.

What is:

$$P(B, e) = (0.1, 0.1, 0.2)$$

$$P(B|e) = \frac{P(B, e)}{P(e)} = ??$$

Normalization

Consider the joint probability table $P(A, B, C)$:

		B		
		b_1	b_2	b_3
A	a_1	$(0, 0.05, 0.05)$	$(0.05, 0.05, 0)$	$(0.05, 0.05, 0.05)$
	a_2	$(0.1, 0.1, 0)$	$(0.1, 0, 0.1)$	$(0.2, 0, 0.05)$

Assume evidence e: $A = a_2$ and $C = c_1$.

What is:

$$P(B, e) = (0.1, 0.1, 0.2)$$

$$P(B|e) = \frac{P(B, e)}{P(e)} = \frac{(0.1, 0.1, 0.2)}{0.4} = (0.25, 0.25, 0.5)$$

Conditional independence

A is independent of B:

- Information on B does not change my belief in A.

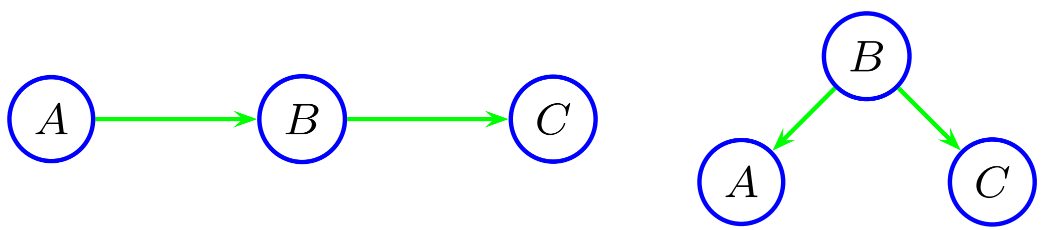
$$P(A|B) = P(A)$$

In the context c, $P(A|B,c) = P(A|c)$:

- A is independent of B given c.

If the state of B is known then A is independent of C.

$$P(A|B,C) = P(A|B)$$



Conditional independence is symmetric:

$$P(A|B,C) = P(A|B) \Leftrightarrow P(C|A,B) = P(C|B).$$

Conditional independence: An example

Turnover in % (type, weekday,shop)

		Type		
		Shirts	Jeans	Underwear
Weekday	Mo	$(2.52, 1.05, 0.63)$	$(3.24, 1.35, 0.81)$	$(1.44, 0.60, 0.36)$
	Tu	$(3.57, 1.49, 0.89)$	$(4.59, 1.91, 1.15)$	$(2.04, 0.85, 0.51)$
	We	$(3.99, 1.66, 1.00)$	$(5.13, 2.14, 1.28)$	$(2.28, 0.95, 0.57)$
	Th	$(4.20, 1.75, 1.05)$	$(5.40, 2.25, 1.35)$	$(2.40, 1.00, 0.60)$
	Fr	$(4.62, 1.93, 1.65)$	$(5.94, 2.47, 1.49)$	$(2.64, 1.10, 0.66)$
	Sa	$(2.70, 0.87, 0.53)$	$(2.70, 1.13, 0.67)$	$(1.20, 0.50, 0.30)$

$P(\text{Shop}|\text{Shirts}, \text{Mo}) = ??$

$P(\text{Shop}|\text{Mo}) = ??$

Conditional independence: An example

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		Type		
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	Tu	$(3.57, 1.49, 0.89)$	$(4.59, 1.91, 1.15)$	$(2.04, 0.85, 0.51)$
	We	$(3.99, 1.66, 1.00)$	$(5.13, 2.14, 1.28)$	$(2.28, 0.95, 0.57)$
	Th	$(4.20, 1.75, 1.05)$	$(5.40, 2.25, 1.35)$	$(2.40, 1.00, 0.60)$
	Fr	$(4.62, 1.93, 1.65)$	$(5.94, 2.47, 1.49)$	$(2.64, 1.10, 0.66)$
	Sa	$(2.70, 0.87, 0.53)$	$(2.70, 1.13, 0.67)$	$(1.20, 0.50, 0.30)$

$$P(\text{Shop}|\text{Shirts}, \text{Mo}) = \frac{(2.52, 1.05, 0.63)}{2.52 + 1.05 + 0.63} = (0.6, 0.25, 0.15)$$

$$P(\text{Shop}|\text{Mo}) = ??$$

Conditional independence: An example

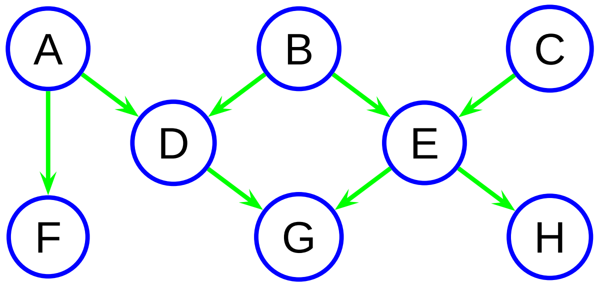
Turnover in % (type, weekday,shop)

		Type		
		Shirts	Jeans	Underwear
Weekday	Mo	$(2.52, 1.05, 0.63)$	$(3.24, 1.35, 0.81)$	$(1.44, 0.60, 0.36)$
	Tu	$(3.57, 1.49, 0.89)$	$(4.59, 1.91, 1.15)$	$(2.04, 0.85, 0.51)$
	We	$(3.99, 1.66, 1.00)$	$(5.13, 2.14, 1.28)$	$(2.28, 0.95, 0.57)$
	Th	$(4.20, 1.75, 1.05)$	$(5.40, 2.25, 1.35)$	$(2.40, 1.00, 0.60)$
	Fr	$(4.62, 1.93, 1.65)$	$(5.94, 2.47, 1.49)$	$(2.64, 1.10, 0.66)$
	Sa	$(2.70, 0.87, 0.53)$	$(2.70, 1.13, 0.67)$	$(1.20, 0.50, 0.30)$

$$P(\text{Shop}|\text{Shirts}, \text{Mo}) = \frac{(2.52, 1.05, 0.63)}{2.52 + 1.05 + 0.63} = (0.6, 0.25, 0.15)$$

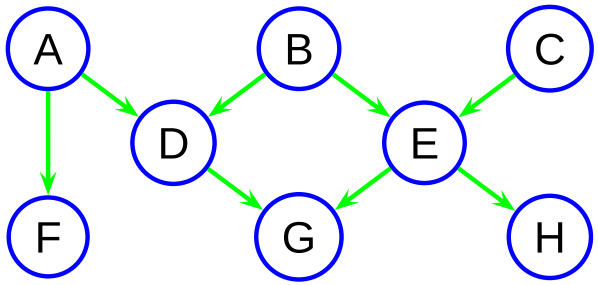
$$P(\text{Shop}|\text{Mo}) = \frac{(2.52 + 3.24 + 1.44, 1.05 + 1.35 + 0.6, 0.63 + 0.81 + 0.36)}{12} = (0.6, 0.25, 0.15)$$

Bayesian belief updating



Find $P(B|a, f, g, h)$

Bayesian belief updating



Find $P(B|a, f, g, h)$

We can if we have access to $P(a, B, C, D, E, f, g, h)$:

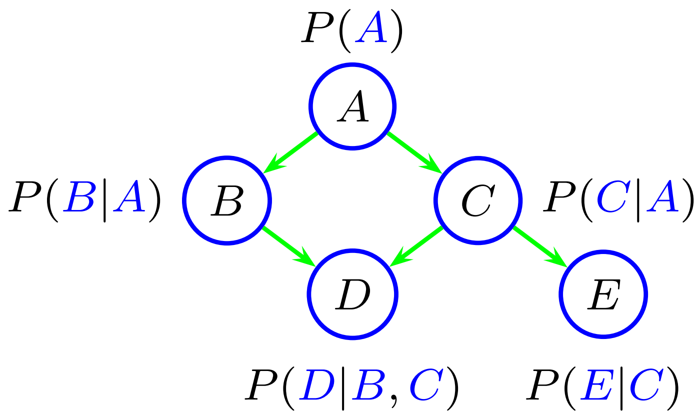
$$P(B, a, f, g, h) = \sum_{C,D,E} P(a, B, C, D, E, f, g, h)$$

$$P(B|a, f, g, h) = \frac{P(B, a, f, g, h)}{P(a, f, g, h)},$$

where

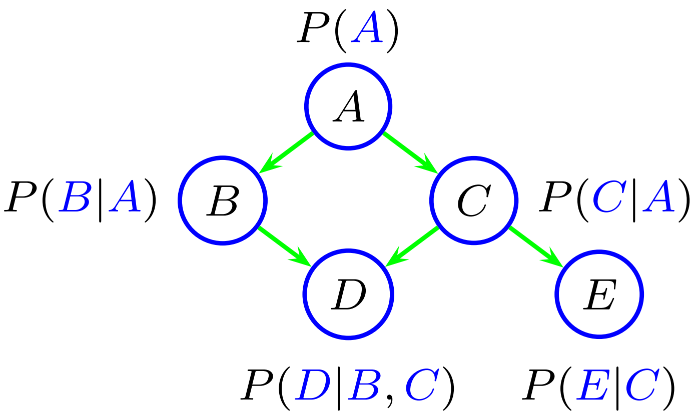
$$P(a, f, g, h) = \sum_{B} P(B, a, f, g, h)$$

Joint probabilities



Calculate $P(A, B, C, D, E)$

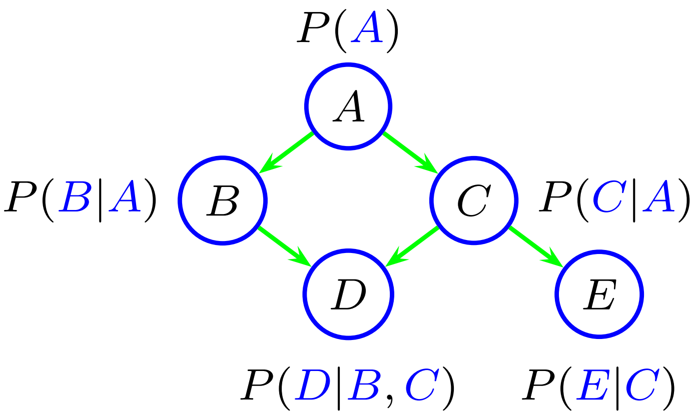
Joint probabilities



Calculate $P(A, B, C, D, E)$

$$P(A, B, C, D, E) = P(E|A, B, C, D)P(A, B, C, D)$$

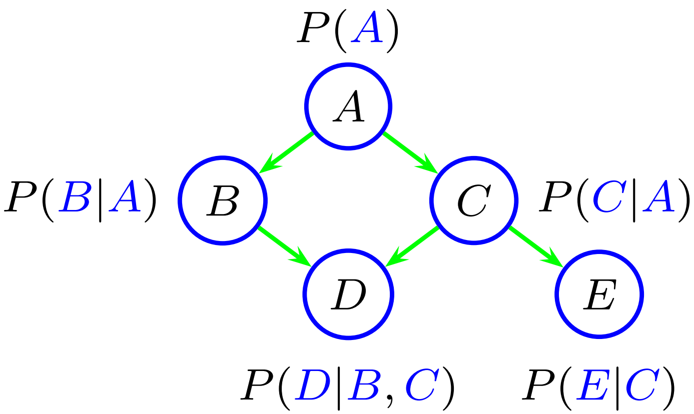
Joint probabilities



Calculate $P(A, B, C, D, E)$

$$\begin{aligned} P(A, B, C, D, E) &= P(E|A, B, C, D)P(A, B, C, D) \\ &= P(E|C)P(A, B, C, D) \end{aligned}$$

Joint probabilities



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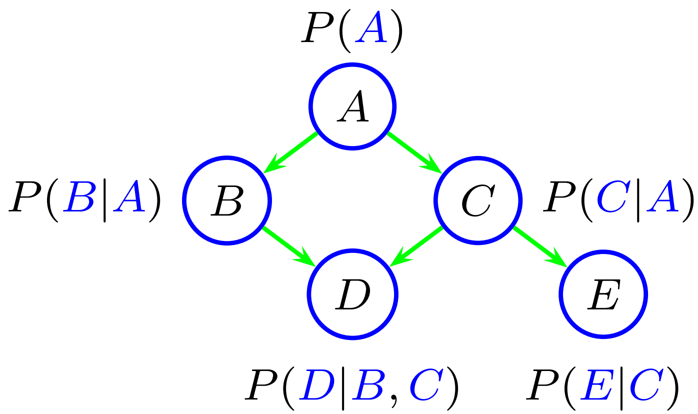
Joint probabilities



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$$\begin{aligned}
P(A,B,C,D,E) &= P(E|A,B,C,D)P(A,B,C,D) \\
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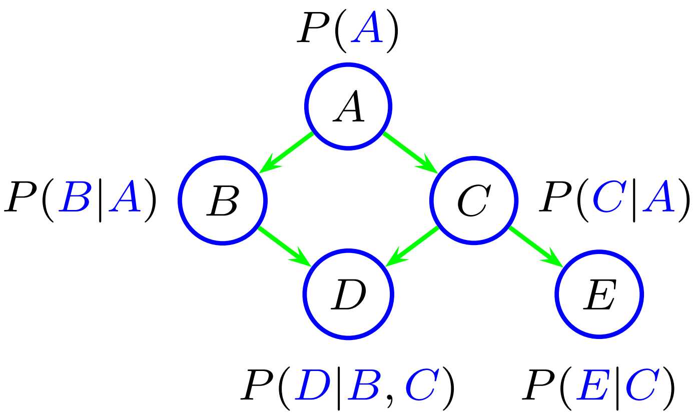
\end{aligned}$$

Joint probabilities



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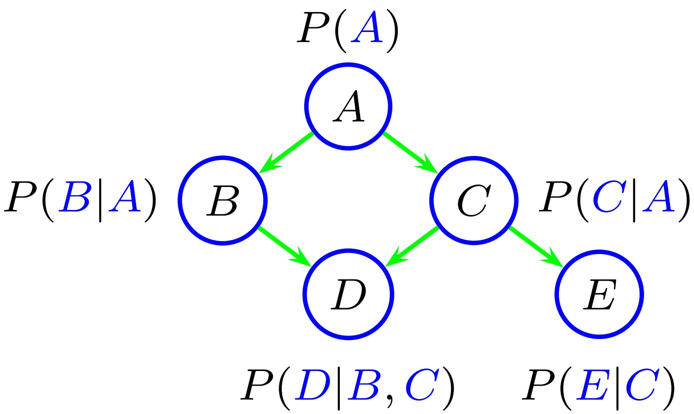
&= P(E|C)P(D|B, C)P(C|A, B)P(A, B)
\end{aligned}
$$

Joint probabilities



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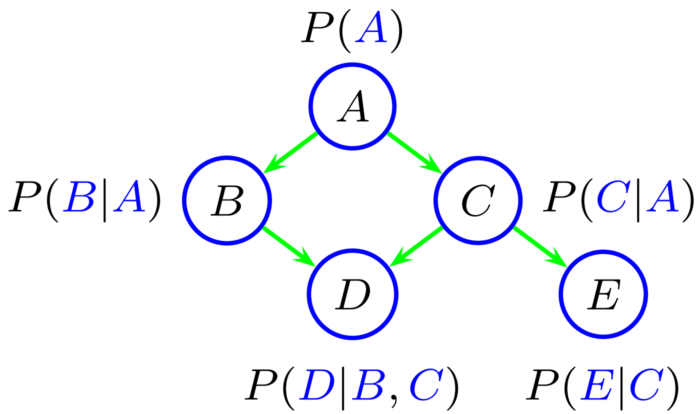
&= P(E|C)P(D|B, C)P(C|A, B)P(A, B) \\
&= P(E|C)P(D|B, C)P(C|A)P(B, A)
\end{aligned}$$

Joint probabilities



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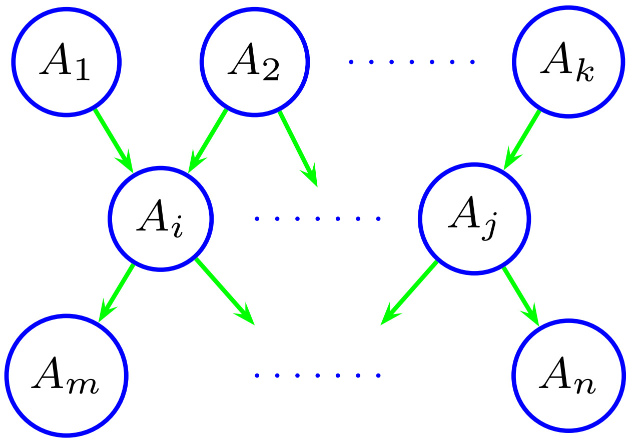
&= P(E|C)P(D|B,C)P(C|A)P(B,A) \\
&= P(E|C)P(D|B,C)P(C|A)P(B|A)P(A)
\end{aligned}
$$

The chain rule



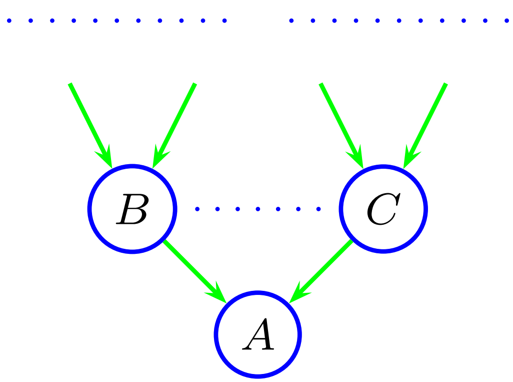
Let BN be a Bayesian network over $\mathcal{U} = \{A_1, \ldots, A_n\}$

Then:

$$P(\mathcal{U}) = \prod_i P(A_i|\text{Pa}(A_i)),$$

where Pa(A_i) are the parents of A_i.

- $P(\mathcal{U})$ is the product of the potentials specified in BN.
- BN is a compact representation of $P(\mathcal{U})$.



$$P(\mathcal{U}) = P(A|\mathcal{U} \setminus \{A\})P(\mathcal{U} \setminus \{A\})$$

$$= P(A|B, \ldots, C) \prod_{X \in \mathcal{U} \setminus \{A\}} P(X|\text{Pa}(X))$$

Evidence I

Consider a variable A with five states a_1, a_2, a_3, a_4, a_5 and with probability:

$$P(A) = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix}, \sum_{i=1}^{5} x_i = 1$$

Assume that we get the evidence e: "A is either in state a_2 or a_4". Then:

$$P(A, e) = \begin{pmatrix} 0 \\ x_2 \\ 0 \\ x_4 \\ 0 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

Thus, e can be represented by a potential $\bar{e} = (0, 1, 0, 1, 0)^T$ and:

$$P(A, e) = P(A) \cdot \bar{e}$$

Evidence II

Definition: Let A be a variable with n states. A finding on A is an n-dimensional table with 0s and 1s.

Semantics: The states marked with a 0 are impossible.

Theorem: Let BN be a Bayesian network over the universe $\mathcal{U} = \{A_1, \ldots, A_n\}$, and let $\bar{e}_1$, $\bar{e}_2$,. . . , $\bar{e}_m$ be findings. Then:

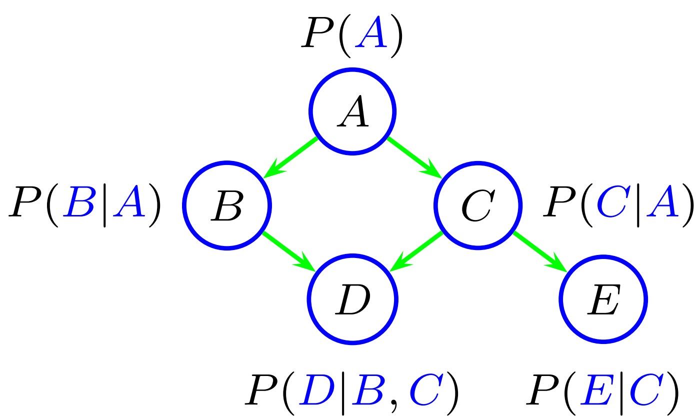
$$P(\mathcal{U}, e) = P(\mathcal{U}) \cdot \prod_{i=1}^{m} \bar{e}_i$$

$$= \prod_{i=1}^{n} P(A_i|\text{Pa}(A_i)) \prod_{j=1}^{m} \bar{e}_j .$$

Hence, to find $P(A|e)$ we use:

$$P(A|e) = \frac{\sum_{\mathcal{U}\setminus\{A\}} P(\mathcal{U}, e)}{P(e)} .$$

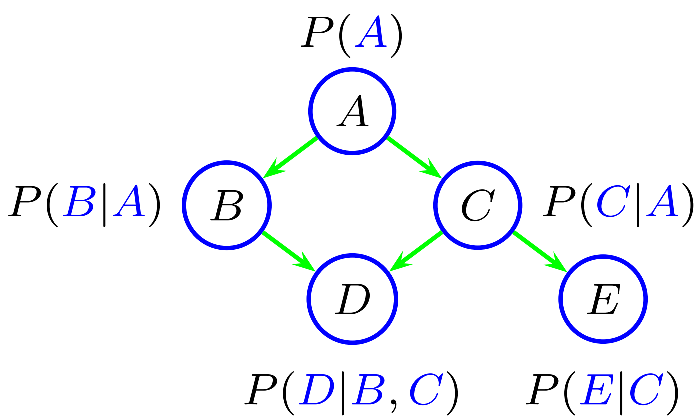
Variable elimination



Do we need $P(\mathcal{U}) = P(A, B, C, D, E)$ in order to calculate $P(A|c, e)$?

Note: $P(A|c, e) = \frac{\sum_B \sum_D P(A,B,c,D,e)}{P(c,e)}$.

Variable elimination



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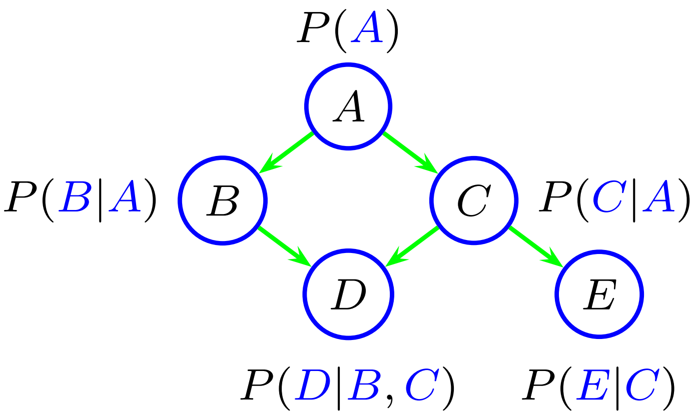
$$\sum_B \sum_D P(A,B,c,D,e) = \sum_B \sum_D P(e|c)P(c|A)P(D|c,B)P(A)P(B|A)$$

Variable elimination



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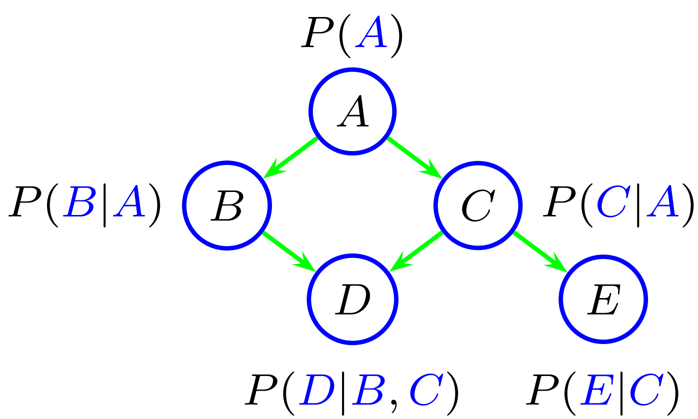
Variable elimination



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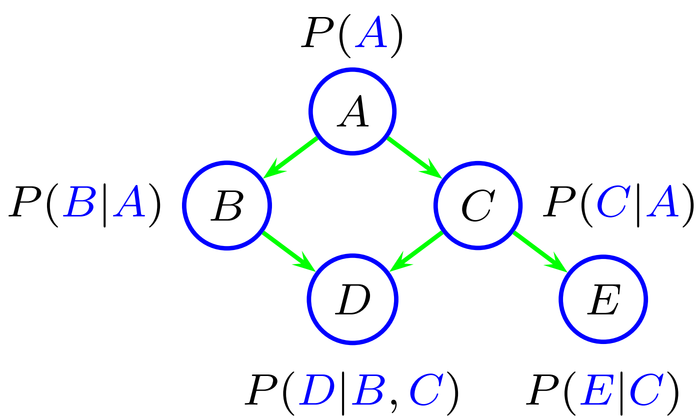
Variable elimination



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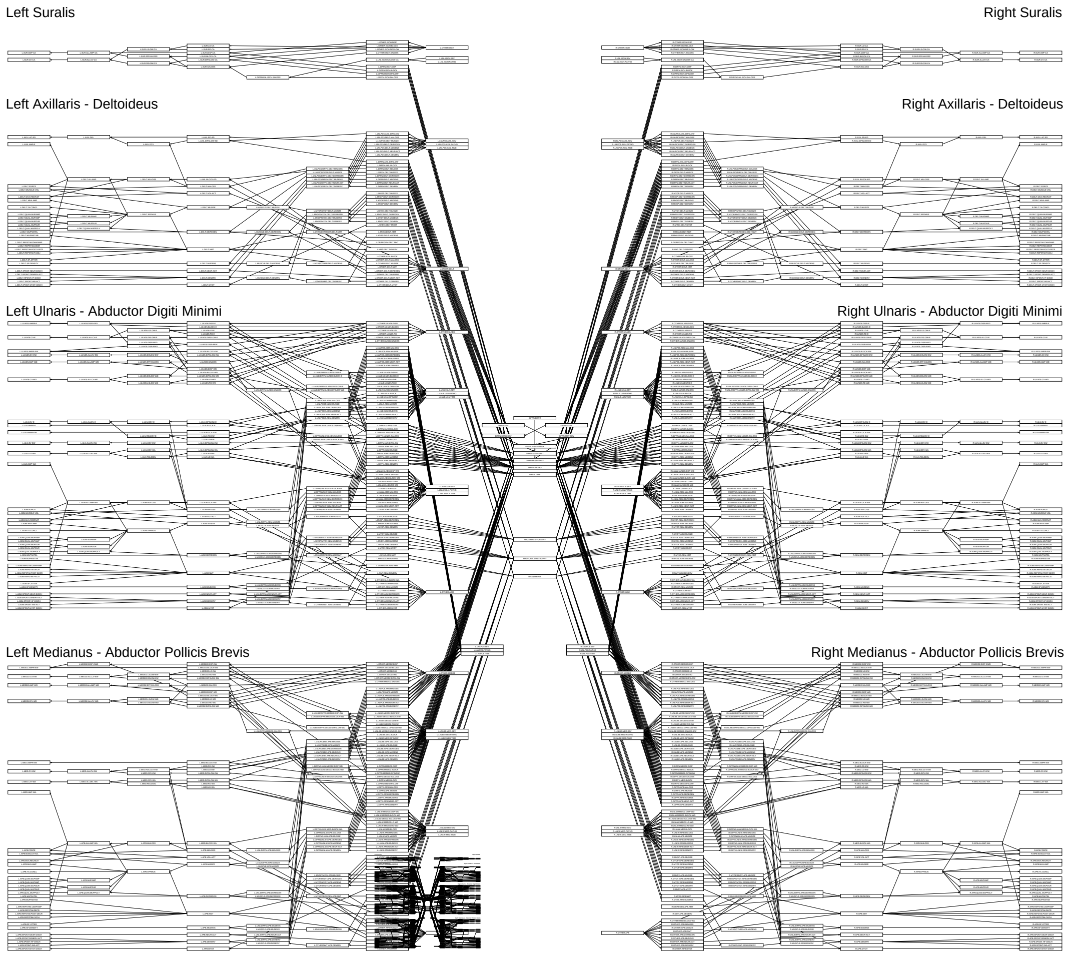
$$\begin{aligned}
\sum_B \sum_D P(A,B,c,D,e) &= \sum_B \sum_D P(e|c)P(c|A)P(D|c,B)P(A)P(B|A) \\
&= P(e|c)P(c|A)P(A) \sum_B \sum_D P(D|c,B)P(B|A) \\
&= P(e|c)P(c|A)P(A) \sum_B P(B|A) \sum_D P(D|c,B) \\
&= P(e|c)P(c|A)P(A)
\end{aligned}$$

So instead of constructing a table with 2^5 entries we only need 2 numbers!

The Munin network



Characteristics:

- Approximately 1100 variables.
- Each variable has between 2 and 20 states.
- 10^{600} possible state configurations!

A system for diagnosing neuro-muscular diseases.

Graphical models

Bayesian networks is one example of graphical models:

Qualitative part: A graph with nodes and links.

Quantitative part: A set of potentials.

Pros:

- Graphs are excellent for inter-human communication.
- Graphical models can be given a sufficient formal semantics.
- Graphical models can be given a formal semantics so that they can be "read" by computers.

Problems:

- The scope of the models.
- The computation task may be intractable.

Summary

What have we considered today?

- Conditional independence:
  - A and C are independent given B if $P(A|C,B) = P(A|B)$.

  This is related to serial and diverging connections.
- Bayesian networks:
  - A directed acyclic graph, where there for each node A with parents $B_1, \ldots, B_n$ is attached a conditional probability table $P(A|B_1, \ldots, B_n)$.
- The chain rule:
  - For a Bayesian network over $\mathcal{U} = \{A_1, \ldots, A_n\}$ we have: $P(\mathcal{U}) = \prod_{i=1}^{n} P(A_i|\text{Pa}(A_i))$.
- Evidence:
  - If $\bar{e}_1, \ldots, \bar{e}_m$ are findings, then:

$$P(\mathcal{U}, e) = \prod_{i=1}^{n} P(A_i|\text{Pa}(A_i)) \prod_{j=1}^{m} \bar{e}_j \qquad P(A|e) = \frac{\sum_{\mathcal{U}\setminus\{A\}} P(\mathcal{U}, e)}{P(e)}.$$