Solutions for Bayesian networks and decision graphs (second edition)

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Solution for exercise 2.1

(i) $\frac{2}{3}, \frac{1}{3}, \frac{1}{2}$.

(ii) Close to zero. Notice that the certainty resulting from the combined action is much smaller than the minimum of the effects of single actions.

Solution for exercise 2.2

The structure is similar to the structure of the "Car Start" network.

Solution for exercise 2.3

See the Hugin network.

Solution for exercise 2.4

In (a), all variables are d-connected to A.
In (b), all variables except C and F are d-connected to A.

Solution for exercise 2.5

Fuel Meter Standing and *Fuel?* cannot be d-separated.

Fuel Meter Standing and *Start?* can be d-separated by {*Fuel?*} and {*Fuel?*,*Clean Spark Plugs*}.

Fuel Meter Standing and *Clean Spark Plugs* can be d-separated by $\emptyset$, {*Fuel?*}, and {*Fuel?*,*Start?*}.

Fuel? and *Start?* cannot be d-separated.

Fuel? and *Clean Spark Plugs* can be d-separated by $\emptyset$ and $\{$*Fuel Meter Standing*$\}$.

Start? and *Clean Spark Plugs* cannot be d-separated.

Solution for exercise 2.6

Minimal sets d-separating C and E: $\{B, D, F\}$ and $\{A, B, D\}$.

Minimal sets d-separating A and B: $\emptyset$.

Maximal set d-separating C and E: $\{A, B, D, F\}$.

Maximal set d-separating A and B: $\{F\}$.

Solution for exercise 2.7

The Markov blanket for

$$\begin{array}{ll} A: & \{B, C, D, F\} \\ B: & \{A, C, D, E, F\} \\ C: & \{A, B, D\} \\ D: & \{A, B, C, E, F\} \\ E: & \{B, D, F\} \\ F: & \{A, B, D, E\} \end{array}$$

Solution for exercise 2.8

Consider a path from A to B, and let C be the neighbor to A on that path. Then C is instantiated. If the C-connection is serial or diverging, then C blocks the path. If the link is converging, consider C's neighbor, D, on the path. As A and D have the common child C, D is in the Markov blanket for A and must be instantiated. Since the direction on the (D, C) link is from D to C, the D-connection on the path from A through C and D to B cannot be converging, and therefore D blocks the path.

Solution for exercise 2.9

The ancestral graph consists of three distinct nodes, A, B, and C. As there is no path between A and B in this graph, A and B are d-separated given C.

Solution for exercise 2.10

(a), (b) and (c) are I-equivalent.

Solution for exercise 2.11

Let $C = \{A_1, ..., A_{i-1}\} \setminus pa(A_i)$, and let $A \in C$. All paths from A to A_i through a parent of A_i are blocked. So it is sufficient to prove that any path from A to A_i through a descendant of A_i is inactive. Take such a path, and let A_j be a the node of maximal index on that path. Then the two neighbors on the path have a lower index, and therefore they must be parents of A_j. As $j > i$ the path cannot be active.

Solution for exercise 2.12

The needed tables are $P(A)$, $P(B)$, $P(C \mid A, B)$, $P(D \mid A, C)$, $P(E \mid B, D, F)$, and $P(F \mid A)$.

Solution for exercise 2.13

A and C are not d-separated given B. A and C are conditionally independent given B.

Solution for exercise 2.14

$P(A, B, C, D, E, F) = P(A)P(B)P(C \mid A, B)P(D \mid A, C)P(E \mid B, D, F)P(F \mid A)$.

For proving that B and D are conditionally independent given A and C, we first calculate

$$\begin{aligned}
P(B, D, a, c) &= \sum_E \sum_F P(a)P(B)P(c \mid a, B)P(D \mid a, c)P(E \mid B, D, F)P(F \mid a) \\
&= P(a)P(B)P(c \mid a, B)P(D \mid a, c) \sum_E \sum_F P(E \mid B, D, F)P(F \mid a) \\
&= P(a)P(B)P(c \mid a, B)P(D \mid a, c) \ .
\end{aligned}$$

Then we use this expression as such

$$\begin{aligned}
P(B|D, a, c) &= \frac{P(a)P(B)P(c \mid a, B)P(D \mid a, c)}{\sum_B P(a)P(B)P(c \mid a, B)P(D \mid a, c)} \\
&= \frac{P(a)P(B)P(c \mid a, B)P(D \mid a, c)}{P(a)P(D \mid a, c) \sum_B P(B)P(c \mid a, B)} \\
&= \frac{P(B)P(c \mid a, B)}{\sum_B P(B)P(c \mid a, B)} \\
&= \frac{P(B, c \mid a)}{\sum_B P(B, c \mid a)} \\
&= P(B \mid a, c) \ .
\end{aligned}$$

That is, we have $P(B|D, a, c) = P(B \mid a, c)$, for the arbitrary choice of a and c, and thus that B and D are conditionally independent given A and C.

Solution for exercise 2.15

Solution for exercise 2.16

Solution for exercise 2.17

From the chain rule we have

$$\begin{aligned} P(B, C, e) &= P(A)P(B \mid A)P(C \mid A, B)e \\ &= P(a_2)P(B \mid a_2)P(C \mid a_2, B) \ . \end{aligned}$$

Plugging the correct values into this expression we get

$$\begin{aligned} P(b_1, c_1, e) &= 0.1 \cdot 0.6 \cdot 0.1 = 0.006 \\ P(b_1, c_2, e) &= 0.1 \cdot 0.6 \cdot 0.9 = 0.054 \\ P(b_2, c_1, e) &= 0.1 \cdot 0.4 \cdot 0.2 = 0.008 \\ P(b_2, c_2, e) &= 0.1 \cdot 0.4 \cdot 0.8 = 0.032 \ . \end{aligned}$$

Solution for exercise 2.18

We have the joint expression

$$P(A, B, C, D, E, F) = P(A)P(B)P(C \mid A, B)P(D \mid A, C)P(E \mid B, D, F)P(F \mid A) \ .$$

Calculating $P(F|c_1)$, we first establish $P(c_1, F)$:

$$\begin{aligned} P(c_1, F) &= \sum_A \sum_B \sum_D \sum_E P(A)P(B)P(c_1 \mid A, B)P(D \mid A, c_1)P(E \mid B, D, F) \\ &\quad P(F \mid A) \\ &= \sum_A P(F \mid A)P(A) \sum_B P(B)P(c_1 \mid A, B) \sum_D P(D \mid A, c_1) \\ &\quad \sum_E P(E \mid B, D, F) \ . \end{aligned}$$

We thus first need to calculate $\sum_E P(E \mid B, D, F)$, which is a table of 10^4 cells. It is reduced to a vacuous potential and dropped from consideration. Similarly, the calculation of $\sum_D P(D \mid A, c_1)$ (which requires manipulation of a table of size 10^2) results in a vacuous potential. Next, $\sum_B P(B)P(c_1 \mid A, B)$ results in a potential ϕ_A over A alone, requiring manipulation of a table of size 10^2 during computation). Finally, $\sum_A P(F \mid A)P(A)\phi_A$ requires manipulation of a table of size 10^2 for computation, and results in the potential $P(c_1, F)$ over F only.

The final step of the computation, where we divide $P(c_1, F)$ with $P(c_1) = \sum_F P(c_1, F)$, requires only manipulation of a table of size 10. The size of the largest potentials we need to manipulate is thus 10^4.

Solution for exercise 2.19

(i) As B is d-separated from C given A we have $P(B|A,C) = P(B|A)$.

(ii) $P(A,B,C) = P(A)P(B|A)P(C|A)$. For example, $P(A, b_1, c_1) = (0.01, 0.162)$.

Solution for exercise 2.21

See the Hugin network.

Solution for exercise 2.22

See the Hugin network.

Solution for exercise 2.23

See the Hugin network.

Solution for exercise 2.24

See the Hugin network.