Solutions for Bayesian networks and decision graphs (second edition)

Finn V. Jensen and Thomas D. Nielsen

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Solution for exercise 6.1

Setting the derivative equal to zero we get the equation

$$\mu 80\theta^{79}(1-\theta)^{20} - \mu\theta^{80}20(1-\theta)^{19} = 0.$$

This holds if $\theta = 0, \theta = 1$ or $4(1-\theta) - \theta = 0$. The latter yields $\theta = 0.8$. As the function is zero for $\theta = 0$ or 1 and positive for $0 < \theta < 1$, it must be maximal for $\theta = 0.8$

Solution for exercise 6.2

(i) 4

(ii) $f_p = \frac{1}{P(pinup)}\theta^2$ for $0 \leq \theta < 1/2$ and $f_p = \frac{1}{P(pinup)}\theta(1-\theta)$ for $1/2 \leq \theta \leq 1$

(iii) 8

(iv) $\frac{7}{12}$

Solution for exercise 6.3

Max. likelihood: $P(T_1) = (0.52, 0.48)$, $P(T_2|a) = (\frac{32}{52}, \frac{20}{52})$, $P(T_2|b) = (\frac{17}{48}, \frac{31}{48})$

Bayesian: $P(T_1) = (\frac{53}{102}, \frac{49}{102})$, $P(T_2|a) = (\frac{33}{54}, \frac{21}{54})$, $P(T_2|b) = (\frac{18}{50}, \frac{32}{50})$

Solution for exercise 6.4

Induction in $n + m$.
The case $n + m = 1$ is basically done in Section 6.1.2.

Assume the theorem to hold for $n + m = i$, and assume that we have $i + 1$ experiments. We can assume wlog that they are performed in a sequence. After the first i experiments we have n "yes" and m "no", and from the induction hypthesis we have a posterior distribution function:

$$f'_p = \mu' \theta^n (1-\theta)^m.$$

If the next experiment is "yes", we get the function

$$f_p = \mu \theta \theta^n (1-\theta)^m = \mu \theta^{n+1} (1-\theta)^m.$$

If the next experiment is "no", we get the function

$$f_p = \mu (1-\theta) \theta^n (1-\theta)^m = \mu \theta^n (1-\theta)^{m+1}.$$

Solution for exercise 6.5

0.36.

Solution for exercise 6.6 The pattern of missingness is (apparently) only dependent on the state of A, hence the data is missing at random (MAR)

Solution for exercise 6.7

The data should be missing complete at random (MCAR). This can also be deduced from exercise 6.6 together with the conclusion reached in the beginning of Section 6.2.

Solution for exercise 6.8

Part 1:

Using the initial Bayesian network we get:

$$E[N(Pr)] = (0.5 + 1 + 1 + 1 + 0.5, 0.5 + 0 + 0 + 0 + 0.5) = (4, 1)$$

$E[N(Pr, UT)] =$

	pos	neg
pos	0.5+1+0.5+0+0.25	0+0+0.5+1+0.25
neg	0.5+0+0+0+0.25	0+0+0+0+0.25

$=$

	pos	neg
pos	2.25	1.75
neg	0.75	0.25

$E[N(Pr, BT)]$ =

	pos	neg
pos	0.5+0+1+1+0	0+1+0+0+0.5
neg	0.5+0+0+0+0	0+0+0+0+0.5

=

	pos	neg
pos	2.5	1.5
neg	0.5	0.5

From these expected counts we find new (maximum likelihood) estimates for the probability parameters:

$$P(Pr) = (4/5, 1/5) = (0.8, 0.2)$$

$P(UT|Pr)$ =

	pos	neg
pos	2.25/4	1.75/4
neg	0.75 /1	0.25/1

=

	pos	neg
pos	0.5625	0.4375
neg	0.75	0.25

$P(BT|Pr)$ =

	pos	neg
pos	2.5/4	1.5/4
neg	0.5 /1	0.5/1

=

	pos	neg
pos	0.625	0.375
neg	0.5	0.5

Part 2:

Using the network generated in the first iteration of the EM algorithm, we calculate new expected counts for the three families of variables in the network.

$$E[N(Pr)] = 0.79 + 1 + 1 + 1 + .75, 0.21 + 0 + 0 + 0 + 0.25) = (4.54, 0.46)$$

$E[N(Pr, UT)]$ =

	pos	neg
pos	0.79+1+0.5625+0+0.4219	0+0+0.4375+1+0.3281
neg	0.21+0+0+0+0.1875	0+0+0+0+0.0625

=

	pos	neg
pos	2.7744	1.7656
neg	0.3975	0.0625

$E[N(Pr, BT)]$ =

	pos	neg
pos	0.79+0+1+1+0	0+1+0+0+0.75
neg	0.21+0+0+0+0	0+0+0+0+0.25

=

	pos	neg
pos	2.79	1.75
neg	0.21	0.25

As above, we use these expected counts to find new (maximum likelihood) estimates for the probability parameters:

$$P(Pr) = (4.54/5, 0.46/5) = (0.908, 0.092)$$

$P(UT|Pr) =$

	pos	neg
pos	2.7744/4.54	1.7656/4.54
neg	0.3975 /0.46	0.0625/0.46

$=$

	pos	neg
pos	0.61	0.39
neg	0.864	0.136

$P(BT|Pr) =$

	pos	neg
pos	2.79/4.54	1.75/4.54
neg	0.21 /0.46	0.25/.46

$=$

	pos	neg
pos	0.615	0.385
neg	0.457	0.543

Solution for exercise 6.9

After the first iteration we have $P(BT = pos|Pr = yes) = \frac{5}{8}$ and $P(BT = pos|Pr = no) = 0.5$.

After the second iteration we have

$$\begin{aligned} P(Pr = yes) &= \frac{4.62}{5} = 0.92, \\ P(UT = pos|Pr = yes) &= \frac{2.85}{4.62} = 0.62, \\ P(UT = pos|Pr = no) &= \frac{0.24}{0.38} = 0.63, \\ P(BT = pos|Pr = yes) &= \frac{2.87}{4.62} = 0.62, \\ P(UT = pos|Pr = no) &= \frac{0.13}{0.38} = 0.34. \end{aligned}$$

Using only complete cases and max. likelihood we have

$$\begin{aligned} P(Pr = yes) &= 1, \\ P(BT = pos|Pr = yes) &= \frac{2}{3}, \\ P(BT = pos|Pr = no) &= ?, \\ P(UT = pos|Pr = yes) &= \frac{1}{2}, \\ P(UT = pos|Pr = no) &= ?. \end{aligned}$$

With Bayesian estimation we get

$$\begin{aligned} P(Pr = yes) &= \frac{4}{5}, \\ P(BT = pos|Pr = yes) &= \frac{3}{5}, \\ P(BT = pos|Pr = no) &= \frac{1}{2}, \\ P(UT = pos|Pr = yes) &= \frac{2}{4}, \\ P(UT = pos|Pr = no) &= \frac{1}{2}. \end{aligned}$$

Solution for exercise 6.10

By using the network from Exercise 3.28 we get:

(i) $P(\text{Experts}) = (0.043, 0.957)$.

(ii) $P(B = y|A = y) = 0.61$ and $P(B = y|A = n) = 0.4$

Solution for exercise 6.11

(i) Sample size 22, $P(B = y|A = y) = \frac{15}{22} = 0.68$.

(ii) Sample size 10.7, $P(B = y|A = y) = 0.69$.

Solution for exercise 6.12

By a mistake, a parameter too much is mentioned. Ignore the parameter u.

(i) After adaptation, $P(A)$ has the sample sizes $(22\frac{1}{2}, 22\frac{1}{2})$, and $P(B| \neg a)$ has the sample sizes (12, 8).

(ii) The order of the cases is important. If for A the cases come $a, \neg a, a, \neg a, ...$, the sample sizes will stay very close to $(12\frac{1}{2}, 12\frac{1}{2})$. If the cases come with first 10 a's and then 10 $\neg a$'s, the sample sizes will be (11.095, 13.905).

Solution for exercise 6.13

See the Hugin network with parameters after the first step. For this situation we get $P(c) = 0.84$ and $P(T = 1|c) = 0.57$. The formulas in Proposition ?? yield $P(c) = -0.255t + 1$. We get $P(a, c) = 0.36$ and the formulas yield $P(a|c) = -t + 1$. The partial derivative of $P(a|c)$ with respect to t is $\frac{0.745}{(1-0.255t)^2}$.

Solution for exercise 6.14

Solution for exercise 6.15

We have
$P(a,d) = \Sigma_C P(a)P(d|c)\Sigma_B P(C|a,B)P(B) = \frac{5}{40}ts + \frac{5}{40}t + \frac{2}{40}$.
and
$P(\neg a,d) = \Sigma_C P(\neg a)P(d|c)\Sigma_B P(C|\neg a,B)P(B) = \frac{5}{40}s + \frac{7}{40}$
This yields $P(d) = \frac{5}{40}ts + \frac{5}{40}t + \frac{5}{40}s + \frac{9}{40}$.

$\frac{P(a,d)}{P(d)} = 0.8$ yields $ts + t - 4s - 5.2 = 0$. As t and s are in $[0;\ 1]$ there is no solution.

Now, consider $\frac{P(a,d)}{P(d)} = \frac{5ts+5t+2}{5ts+5t+5s+9}$. As a function of t, the fraction is maximal for $t = 1$. The resulting fraction is
$frac5s + 710s + 14 = \frac{1}{2}$